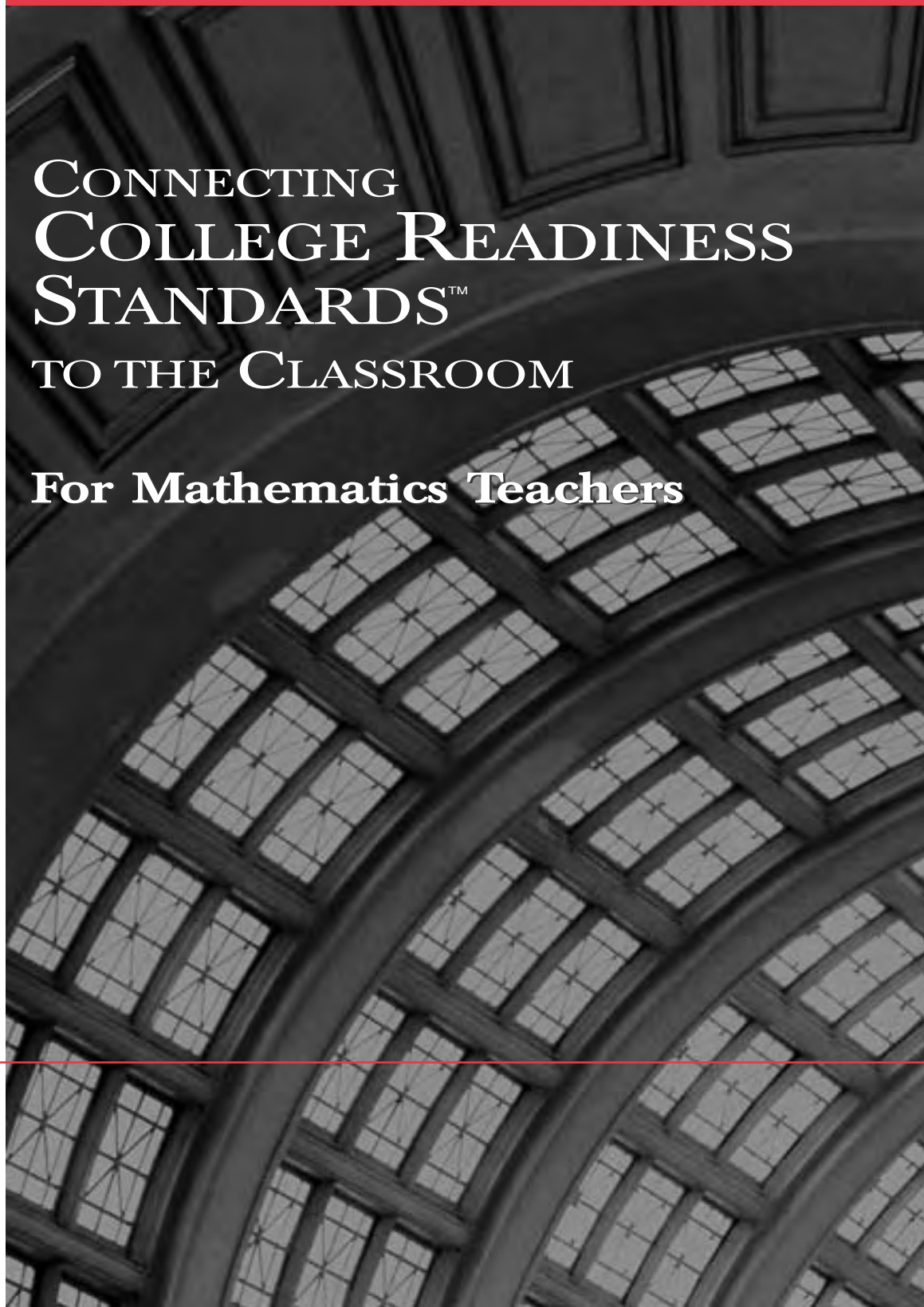
EXPLORE®



Connecting College Readiness Standards™ to the Classroom

For Mathematics Teachers

ACT®

ACT endorses the *Code of Fair Testing Practices in Education* and the *Code of Professional Responsibilities in Educational Measurement*, guides to the conduct of those involved in educational testing. ACT is committed to ensuring that each of its testing programs upholds the guidelines in each *Code*.

A copy of each *Code* may be obtained free of charge from ACT Customer Services (68), P.O. Box 1008, Iowa City, IA 52243-1008, 319/337-1429.

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INTRODUCTION

ACT has developed this guide to help classroom teachers, curriculum coordinators, and counselors interpret the College Readiness Standards™ report for EXPLORE® Mathematics. The guide includes:

- A description of the **College Readiness Standards** for EXPLORE
- A description of the **EXPLORE Mathematics Test**
- A set of **sample test questions**
- A description of the **Assessment-Instruction Link**
- A set of classroom **instructional activities**

The College Readiness Standards for EXPLORE are statements that describe what students who score in the four score ranges 13–15, 16–19, 20–23, and 24–25 are *likely* to know and to be able to do. The statements are generalizations based on the performance of many students scoring in these four score ranges. College Readiness Standards have not been developed for students whose scores fall in the 1–12 range because these students, as a group, do not demonstrate skills similar to each other consistently enough to permit useful generalizations.

The College Readiness Standards for EXPLORE are accompanied by ideas for progress that help teachers identify ways of enhancing student learning based on the scores students receive.

The College Readiness Standards Information Services provide five aggregate reports for EXPLORE. Four of these reports are content specific: each presents the scores of your students in each of the four content areas the EXPLORE test measures—English, Mathematics, Reading, and Science. These four content-specific reports present EXPLORE results using ACT's College Readiness Standards. The fifth report, the Summary Profile, summarizes the scores of your students across all four content areas. All five reports provide data that compare the performance of your school's students with all students in a nationally representative comparison group (norm group).

Local comparisons to the national norm group are most appropriate when EXPLORE is administered under conditions similar to those in the norming study—with all four tests administered in a single session in the standard order, and students having calculators available for use on the Mathematics Test.

Eighth-grade students who test in August through January will receive Fall Eighth-Grade Norms. Eighth graders who test in February through July will receive Spring Eighth-Grade Norms. Ninth-grade students will receive Ninth-Grade Norms regardless of their test date. (If your school chooses to test ninth-grade students in the spring, keep in mind that these students will have had several more months of instruction than the norm group. Therefore, spring-tested ninth graders may show higher levels of achievement when compared to the fall-tested norm group than if they had tested in the fall.) Students who are not in the eighth or ninth grade when they take EXPLORE will receive Fall Eighth-Grade Norms on their student reports.

EXPLORE is a curriculum-based assessment program developed by ACT to help eighth and ninth graders devise a high school course work plan that prepares them to achieve their post-high school goals. As part of ACT's Educational Planning and Assessment System (EPAS™), EXPLORE is complemented by PLAN®, ACT's tenth-grade program, and by the ACT®, for eleventh and twelfth graders. We hope this guide helps you assist your students as they plan and pursue their future studies.

"The role of standardized testing is to let parents, students, and institutions know what students are ready to learn next."

— Ralph Tyler, October 1991
Chairman Emeritus of
ACT's Board of Trustees

The College Readiness Standards Report for EXPLORE Mathematics

The College Readiness Standards report for EXPLORE Mathematics allows you to compare the performance of students in your school with the performance of students nationwide. The report provides summary information you can use to map the development of your students' knowledge and skills in mathematics. Used along with your own classroom observations and with other resources, the test results can help you to analyze your students' progress in mathematics and to identify areas of strength and areas that need more attention. You can then use the Standards as one source of information in the instructional planning process.

A sample report appears on the next page. An explanation of its features is provided below.

A This section briefly explains the uses of the report to help you interpret the test results.

B These are the five score ranges reported for the College Readiness Standards for EXPLORE. To determine the number of score ranges and the width of each score range, ACT staff reviewed normative data, college admission criteria, and information obtained through ACT's Course Placement Service. For a more detailed explanation of the way the score ranges were determined, see page 5.

C This section compares the percent of students who scored in a particular score range at an individual school (Local) with the percent of all students in the norm group who scored in the same range. The percent of students for the norm group is based on the most current set of nationally representative norms. The number of local-school students who scored in each of the five score ranges is provided in the column to the left of each bar graph; the total number of students tested locally is provided at the top of the report.

D The College Readiness Standards were developed by identifying the knowledge and skills students need in order to respond successfully to questions on the EXPLORE Mathematics Test. As you review the report for EXPLORE Mathematics, you will note that the Standards are cumulative, which means that if students score, for example, in the 16–19 score range, they are likely to be able to demonstrate most or all of the knowledge and skills in the 13–15 and the 16–19 score ranges. Students may be able to demonstrate some of the skills in the next score range, 20–23, but not consistently enough as a group to reach that score range. A description of the way the College Readiness Standards were developed can be found on pages 5–6.

E The "ideas for progress" are statements that provide suggestions for learning experiences that students might benefit from. These ideas for progress are arranged by score range and strand. Although many of the ideas cross more than one strand, a primary strand has been identified for each in order to facilitate their use in the classroom. Ideas for progress are provided for students who score in the 24–25 score range, the highest score range for EXPLORE. The ideas for the 24–25 score range are shown to suggest educational experiences from which students may benefit before they take PLAN and the ACT.

F Page 2 of the report profiles the test results, College Readiness Standards, and ideas for progress for score ranges 20–23 and 24–25.

ACT®

College Readiness Standards
Information Services

EXPLORE® Mathematics Report

The College Readiness Standards report for EXPLORE Mathematics allows you to compare the performance of students in your school with the performance of students nationwide. For an explanation of the report's features, see page 2 in the Mathematics guide *Connecting College Readiness Standards to the Classroom*.

A

Sample School (00000000)
Standard Report
Anytown, US

Number of Students: 321
Grade: 8
2005-2006 Academic Year

Score Range	No. of Students	Percentage Local	Percentage National		Basic Operations & Applications	Probability, Statistics, & Data Analysis	Numbers: Concepts & Properties	Expressions, Equations, & Inequalities	Graphical Representations	Properties of Plane Figures	Measurement
1–12	51			Standards	■ Students who score in the 1–12 range are most likely beginning to develop the knowledge and skills assessed in the other score ranges.						
				Ideas for progressing to 13–15 score range	■ practice and apply estimation and computation using whole numbers and decimals ■ choose the appropriate method of computation to solve multistep problems (e.g., calculator, mental, or pencil and paper) ■ practice selecting appropriate units of measure (e.g., inches or feet, hours or minutes, centimeters or meters) and converting between units ■ model and connect physical, verbal, and symbolic representations of money	■ interpret data from a variety of displays and use it in computation (e.g., mean, median, mode, range) ■ organize, display, and analyze data in a variety of ways		■ model a variety of problem situations with expressions and/or equations ■ use the inverse relationships for the basic operations of addition and subtraction to determine unknown quantities	■ locate and describe points in terms of their position on the number line		■ identify line segments in geometric figures and estimate or calculate their measure
13–15	90	26%	86%	Standards	■ Perform one-operation computation with whole numbers and decimals ■ Solve problems in one or two steps using whole numbers ■ Perform common conversions (e.g., inches to feet or hours to minutes)	■ Calculate the average of a list of positive whole numbers ■ Perform a single computation using information from a table or chart	■ Recognize equivalent fractions and fractions in lowest terms	■ Exhibit knowledge of basic expressions (e.g., identify an expression for a total as $b + g$) ■ Solve equations in the form $x + a = b$, where a and b are whole numbers or decimals	■ Identify the location of a point with a positive coordinate on the number line		■ Estimate or calculate the length of a line segment based on other lengths given on a geometric figure
				Ideas for progressing to 16–19 score range	■ investigate and build understanding of the concept of percentage as a comparison of a part to a whole ■ use multiple operations to solve multistep arithmetic problems	■ solve real-world problems that involve measures of central tendency (e.g., mean, median, mode) ■ interpret data from a variety of displays (e.g., box-and-whisker plot) and use it along with additional information to solve real-world problems ■ conduct simple probability experiments and represent results using different formats	■ recognize and apply place value, rounding, and elementary number theory concepts	■ use mathematical symbols and variables to express a relationship between quantities (e.g., the number of 59¢ candy bars that you can buy for $5 must satisfy $59n \leq 500$) ■ evaluate algebraic expressions and solve simple equations using integers	■ locate and describe objects in terms of their position on the number line and on a grid	■ describe, compare, and contrast plane and solid figures using their attributes	■ distinguish between area and perimeter, and find the area or perimeter when all relevant dimensions are given
16–19	152	39%	28%	Standards	■ Solve routine one-step arithmetic problems (using whole numbers, fractions, and decimals) such as single-step percent ■ Solve some routine two-step arithmetic problems	■ Calculate the average of a list of numbers ■ Calculate the average, given the number of data values and the sum of the data values ■ Read tables and graphs ■ Perform computations on data from tables and graphs ■ Use the relationship between the probability of an event and the probability of its complement	■ Recognize one-digit factors of a number ■ Identify a digit's place value	■ Substitute whole numbers for unknown quantities to evaluate expressions ■ Solve one-step equations having integer or decimal answers ■ Combine like terms (e.g., $2x + 5x$)	■ Locate points on the number line and in the first quadrant	■ Exhibit some knowledge of the angles associated with parallel lines	■ Compute the perimeter of polygons when all side lengths are given ■ Compute the area of rectangles when whole number dimensions are given
				Ideas for progressing to 20–23 score range	■ solve routine arithmetic problems that involve rates, proportions, and percents ■ model and solve problems that contain verbal and symbolic representations of money ■ do multistep computations with rational numbers	■ interpret data and use appropriate measures of central tendency to find unknown values ■ find the probability of a simple event in a variety of settings ■ gather, organize, display, and analyze data in a variety of ways to use in problem solving ■ conduct simple probability experiments, use a variety of counting techniques (e.g., Venn diagrams, Fundamental Counting Principle, organized lists), and represent results from data using different formats	■ apply elementary number concepts, including identifying patterns pictorially and numerically (e.g., triangular numbers, arithmetic and geometric sequences), ordering numbers, and factoring ■ recognize, identify, and apply field axioms (e.g., commutative)	■ create expressions that model mathematical situations using combinations of symbols and numbers ■ evaluate algebraic expressions and solve multistep first-degree equations	■ sketch and identify line segments, midpoints, intersections, and vertical and horizontal lines	■ describe angles and triangles using mathematical terminology and apply their properties	■ find area and perimeter of a variety of polygons by substituting given values into standard geometric formulas
20–23	25	8%	4%	Standards	■ Solve routine two-step or three-step arithmetic problems involving concepts such as rate and proportion, tax added, percentage off, and computing with a given average	■ Calculate the missing data value, given the average and all data values but one ■ Translate from one representation of data to another (e.g., a bar graph to a circle graph) ■ Determine the probability of a simple event	■ Exhibit knowledge of elementary number concepts including rounding, the ordering of decimals, pattern identification, absolute value, primes, and greatest common factor	■ Evaluate algebraic expressions by substituting integers for unknown quantities ■ Add and subtract simple algebraic expressions ■ Solve routine first-degree equations ■ Perform straightforward word-to-symbol translations	■ Locate points in the coordinate plane	■ Find the measure of an angle using properties of parallel lines ■ Exhibit knowledge of basic angle properties and special sums of angle measures (e.g., 90°, 180°, and 360°)	■ Compute the area and perimeter of triangles and rectangles in simple problems ■ Use geometric formulas when all necessary information is given
				Ideas for progressing to 24–25 score range	■ apply and use number properties to model and solve problems that involve reasoning with proportions ■ select and use appropriate units when solving problems that involve one or more units of measure	■ construct and analyze Venn diagrams to help determine simple probabilities	■ use the inverse relationships for the four basic operations, exponentiation, and root extractions to determine unknown quantities	■ identify, interpret, and generate symbolic representations that model the context of a problem ■ factor and perform the basic operations on polynomials ■ create and solve linear equations and inequalities that model real-world situations ■ solve literal equations for any variable	■ represent and interpret relationships defined by equations and formulas; translate between representations as ordered pairs, graphs, and equations; and investigate symmetry and transformations (e.g., reflections, translations, rotations)	■ recognize what geometric properties and relationships for parallel lines to apply to find unknown angle measures ■ recognize when to apply geometric properties and relationships of triangles to find unknown angle measures	■ apply a variety of strategies to determine the circumference or perimeter and the area for circles, triangles, rectangles, and composite geometric figures
24–25	3	1%	1%	Standards	■ Solve multistep arithmetic problems that involve planning or converting units of measure (e.g., feet per second to miles per hour)	■ Calculate the average, given the frequency counts of all the data values ■ Manipulate data from tables and graphs ■ Compute straightforward probabilities for common situations	■ Find and use the least common multiple ■ Order fractions ■ Work with numerical factors ■ Work with scientific notation ■ Work with squares and square roots of numbers	■ Solve real-world problems using first-degree equations ■ Write expressions, equations, or inequalities with a single variable for common pre-algebra settings (e.g., rate and distance problems and problems that can be solved by using proportions) ■ Identify solutions to simple quadratic equations		■ Use several angle properties to find an unknown angle measure	■ Compute the area of triangles and rectangles when one or more additional simple steps are required ■ Compute the area and circumference of circles after identifying necessary information
				Ideas for progressing to 28–32 score range (PLAN)	■ model and solve real-world problems that involve a combination of rates, proportions, and/or percents	■ find the probability of simple events, disjoint events, compound events, and independent events in a variety of settings using a variety of counting techniques	■ apply and use elementary number concepts and number properties to model and solve nonroutine problems that involve new ideas	■ create and use basic families of functions (which include linear, absolute value, and quadratic) to model and solve problems in common settings ■ explore and use different methods to solve systems of equations ■ manipulate radical expressions (e.g., rationalize denominators)	■ graph linear equations and inequalities, determine slopes of lines, identify parallel and perpendicular lines, and find distances ■ identify characteristics of figures from a general equation	■ apply special right-triangle properties and the Pythagorean theorem to solve congruent and similar shape problems	■ apply a variety of strategies using relationships between perimeter, area, and volume to calculate desired measures

B **C** **D** **E** **F**

continued

Description of the College Readiness Standards

What Are the College Readiness Standards?

The College Readiness Standards communicate educational expectations. Each Standard describes what students who score in the designated range are *likely* to be able to do with what they know. Students can typically demonstrate the skills and knowledge within the score ranges preceding the range in which they scored, so the College Readiness Standards are cumulative.

In helping students make the transition to high school, teachers, counselors, and parents can use the College Readiness Standards for EXPLORE to interpret students' scores and to understand which skills students need to develop to be better prepared for the future.

How Were the Score Ranges Determined?

To determine the number of score ranges and the width of each score range for EXPLORE, ACT staff both reviewed EXPLORE normative data and considered the relationship among EXPLORE, PLAN, and the ACT.

In reviewing the EXPLORE normative data, ACT staff analyzed the distribution of student scores across the score scale. Because EXPLORE and PLAN have a common score scale, ACT can provide EXPLORE examinees with an estimated PLAN Composite score. When the score ranges were being determined, therefore, both the EXPLORE score scale, 1–25, and the PLAN score scale, 1–32, were reviewed side by side. And because many students take PLAN to determine how well they might perform on the ACT, the course-placement research that ACT has conducted over the last forty years was also reviewed. ACT's Course Placement Service provides colleges and universities with cutoff scores that are used to place students into appropriate entry-level courses in college; and these cutoff scores were used to help define the score ranges.

After analyzing all the data and reviewing different possible score ranges, ACT staff concluded that using the five score ranges 1–12, 13–15, 16–19, 20–23, and 24–25 would best distinguish students' levels of achievement so as to assist teachers, administrators, and others in relating EXPLORE test scores to students' attainment of specific skills and understandings.

How Were the College Readiness Standards Developed?

After reviewing normative data, college admission criteria, and information obtained through ACT's Course Placement Service, content experts wrote the College Readiness Standards based on their analysis of the skills and knowledge students need in order to successfully respond to the test questions in each score range. Experts analyzed numerous test questions that had been answered correctly by 80% or more of the examinees within each score range. The 80% criterion was chosen because it offers those who use the College Readiness Standards a high degree of confidence that students scoring in a given score range will most *likely* be able to demonstrate the skills and knowledge described in that range.

"The examination should describe the student in meaningful terms—meaningful to the student, the parent, and the elementary and high school teacher—meaningful in the sense that the profile scores correspond to recognizable school activities, and directly suggest appropriate distributions of emphasis in learning and teaching."

— E. F. Lindquist, February 1958
Cofounder of ACT

As a content validity check, ACT invited nationally recognized scholars from high school and university Mathematics and Education departments to review the College Readiness Standards for the EXPLORE Mathematics Test. These teachers and researchers provided ACT with independent, authoritative reviews of the ways the College Readiness Standards reflect the skills and knowledge students need to successfully respond to the questions on the EXPLORE Mathematics Test.

Because EXPLORE is curriculum based, ACT and independent consultants conduct a review every three to four years to ensure that the knowledge and skills described in the Standards and outlined in the test specifications continue to reflect those being taught in classrooms nationwide.

How Should the College Readiness Standards Be Interpreted and Used?

The College Readiness Standards reflect the progression and complexity of the skills measured in EXPLORE. Because no EXPLORE test form measures all of the skills and knowledge included in the College Readiness Standards, the Standards must be interpreted as skills and knowledge that *most* students who score in a particular score range are *likely* to be able to demonstrate. Since there were relatively few test questions that were answered correctly by 80% or more of the students who scored in the lower score ranges, the Standards in these ranges should be interpreted cautiously. The skills and understandings of students who score in the 1–12 score range may still be evolving. For these students the skills and understandings in the higher score ranges could become their target achievement outcomes.

It is important to recognize that the EXPLORE Test does not measure everything students have learned nor does any test measure everything necessary for students to know to be successful in high school. The EXPLORE Mathematics Test includes questions from a large domain of skills and from areas of knowledge that have been judged important for success in high school and beyond. Thus, the College Readiness Standards should be interpreted in a responsible way that will help students understand what they need to know and do if they are going to make a successful transition to high school. As students choose courses they plan to take in high school, they can use the Standards to identify the skills and knowledge they need to develop to be better prepared for their future. Teachers and curriculum coordinators can use the Standards to learn more about their students' academic strengths and weaknesses and can then modify their instruction and guide students accordingly.

How Are the College Readiness Standards Organized?

As content experts reviewed the test questions connected to each score range, distinct yet overlapping areas of knowledge and skill were identified. For example, there are many types of questions in which students are asked to solve arithmetic problems. Therefore, *Basic Operations & Applications* is one area, or *strand,* within the College Readiness Standards for EXPLORE Mathematics. The other strands are *Probability, Statistics, & Data Analysis; Numbers: Concepts & Properties; Expressions, Equations, & Inequalities; Graphical Representations; Properties of Plane Figures;* and *Measurement.*

The strands provide an organizational framework for the College Readiness Standards statements. As you review the Standards, you will note a progression in complexity within each strand. For example, in the 13–15 range for the Basic Operations & Applications strand, students are able to "solve problems in one or two steps using whole numbers," while in the 24–25 range, students demonstrate that they are able to "solve multistep arithmetic problems that involve planning or converting units of measure (e.g., feet per second to miles per hour)."

The Standards are complemented by brief descriptions of learning experiences from which students might benefit. Based on the College Readiness Standards, these ideas for progress are designed to provide classroom teachers with help for lesson plan development. These ideas, which are given in Table 1, demonstrate one way that information learned from standardized test results can be used to inform classroom instruction.

Because students learn over time and in various contexts, it is important to use a variety of instructional methods and materials to meet students' diverse needs and to help strengthen and build upon their knowledge and skills. The ideas for progress offer teachers a variety of suggestions to foster learning experiences from which students would likely benefit as they move from one level of learning to the next.

Because learning is a complex and individual process, it is especially important to use multiple sources of information—classroom observations and teacher-developed assessment tools, as well as standardized tests—to accurately reflect what each student knows and can do. The Standards and ideas for progress, used in conjunction with classroom-based and curricular resources, help teachers and administrators to guide the whole education of every student.

What Are the EXPLORE Mathematics Test College Readiness Standards?

Table 1 on pages 8–13 suggests links between what students are *likely* to be able to do (the College Readiness Standards) and what learning experiences students would likely benefit from.

The College Readiness Standards are organized both by score range (along the left-hand side) and by strand (across the top). The lack of a College Readiness Standards statement in a score range indicates that there was insufficient evidence with which to determine a descriptor.

The ideas for progress are also arranged by score range and by strand. Although many of the ideas cross more than one strand, a primary strand has been identified for each in order to facilitate their use in the classroom. For example, the statement in the 20–23 score range "apply and use number properties to model and solve problems that involve reasoning with proportions" brings together concepts from several strands, such as Basic Operations & Applications; Numbers: Concepts & Properties; and Expressions, Equations, & Inequalities. However, this idea is primarily linked to the Basic Operations & Applications strand.

As you review the table, you will note that ideas for progress have been provided for the 24–25 score range, the highest score range for EXPLORE. EXPLORE is designed to measure knowledge and skills achieved through the eighth grade. Ideas for progress for the 24–25 score range are shown to provide ideas for educational experiences from which students may benefit before they take PLAN and the ACT.

EXPLORE MATHEMATICS TEST

Table 1: The College Readiness Standards

The Standards describe what students who score in the specified score ranges are *likely* to know and to be able to do. The ideas for progress help teachers identify ways of enhancing students' learning based on the scores students receive.

		Basic Operations & Applications	***Probability, Statistics, & Data Analysis***	***Numbers: Concepts & Properties***
1–12	**Standards**	■ Students who score in the 1–12 range are most likely beginning to develop the knowledge and skills assessed in the other score ranges.		
	ideas for progress	■ practice and apply estimation and computation using whole numbers and decimals ■ choose the appropriate method of computation to solve multistep problems (e.g., calculator, mental, or pencil and paper) ■ practice selecting appropriate units of measure (e.g., inches or feet, hours or minutes, centimeters or meters) and converting between units ■ model and connect physical, verbal, and symbolic representations of money	■ interpret data from a variety of displays and use it in computation (e.g., mean, median, mode, range) ■ organize, display, and analyze data in a variety of ways	
13–15	**Standards**	■ Perform one-operation computation with whole numbers and decimals ■ Solve problems in one or two steps using whole numbers ■ Perform common conversions (e.g., inches to feet or hours to minutes)	■ Calculate the average of a list of positive whole numbers ■ Perform a single computation using information from a table or chart	■ Recognize equivalent fractions and fractions in lowest terms
	ideas for progress	■ investigate and build understanding of the concept of percentage as a comparison of a part to a whole ■ use multiple operations to solve multistep arithmetic problems	■ solve real-world problems that involve measures of central tendency (e.g., mean, median, mode) ■ interpret data from a variety of displays (e.g., box-and-whisker plot) and use it along with additional information to solve real-world problems ■ conduct simple probability experiments and represent results using different formats	■ recognize and apply place value, rounding, and elementary number theory concepts

Expressions, Equations, & Inequalities	*Graphical Representations*	*Properties of Plane Figures*	*Measurement*
■ model a variety of problem situations with expressions and/or equations ■ use the inverse relationships for the basic operations of addition and subtraction to determine unknown quantities	■ locate and describe points in terms of their position on the number line		■ identify line segments in geometric figures and estimate or calculate their measure
■ Exhibit knowledge of basic expressions (e.g., identify an expression for a total as $b + g$) ■ Solve equations in the form $x + a = b$, where a and b are whole numbers or decimals	■ Identify the location of a point with a positive coordinate on the number line		■ Estimate or calculate the length of a line segment based on other lengths given on a geometric figure
■ use mathematical symbols and variables to express a relationship between quantities (e.g., the number of 59¢ candy bars that you can buy for $5 must satisfy $59n \leq 500$) ■ evaluate algebraic expressions and solve simple equations using integers	■ locate and describe objects in terms of their position on the number line and on a grid	■ describe, compare, and contrast plane and solid figures using their attributes	■ distinguish between area and perimeter, and find the area or perimeter when all relevant dimensions are given

EXPLORE MATHEMATICS TEST

Table 1 (continued): The College Readiness Standards

The Standards describe what students who score in the specified score ranges are *likely* to know and to be able to do. The ideas for progress help teachers identify ways of enhancing students' learning based on the scores students receive.

		Basic Operations & Applications	***Probability, Statistics, & Data Analysis***	***Numbers: Concepts & Properties***
16–19	**Standards**	■ Solve routine one-step arithmetic problems (using whole numbers, fractions, and decimals) such as single-step percent ■ Solve some routine two-step arithmetic problems	■ Calculate the average of a list of numbers ■ Calculate the average, given the number of data values and the sum of the data values ■ Read tables and graphs ■ Perform computations on data from tables and graphs ■ Use the relationship between the probability of an event and the probability of its complement	■ Recognize one-digit factors of a number ■ Identify a digit's place value
	ideas for progress	■ solve routine arithmetic problems that involve rates, proportions, and percents ■ model and solve problems that contain verbal and symbolic representations of money ■ do multistep computations with rational numbers	■ interpret data and use appropriate measures of central tendency to find unknown values ■ find the probability of a simple event in a variety of settings ■ gather, organize, display, and analyze data in a variety of ways to use in problem solving ■ conduct simple probability experiments, use a variety of counting techniques (e.g., Venn diagrams, Fundamental Counting Principle, organized lists), and represent results from data using different formats	■ apply elementary number concepts, including identifying patterns pictorially and numerically (e.g., triangular numbers, arithmetic and geometric sequences), ordering numbers, and factoring ■ recognize, identify, and apply field axioms (e.g., commutative)
20–23	**Standards**	■ Solve routine two-step or three-step arithmetic problems involving concepts such as rate and proportion, tax added, percentage off, and computing with a given average	■ Calculate the missing data value, given the average and all data values but one ■ Translate from one representation of data to another (e.g., a bar graph to a circle graph) ■ Determine the probability of a simple event	■ Exhibit knowledge of elementary number concepts including rounding, the ordering of decimals, pattern identification, absolute value, primes, and greatest common factor
	ideas for progress	■ apply and use number properties to model and solve problems that involve reasoning with proportions ■ select and use appropriate units when solving problems that involve one or more units of measure	■ construct and analyze Venn diagrams to help determine simple probabilities	■ use the inverse relationships for the four basic operations, exponentiation, and root extractions to determine unknown quantities

Expressions, Equations, & Inequalities	*Graphical Representations*	*Properties of Plane Figures*	*Measurement*
■ Substitute whole numbers for unknown quantities to evaluate expressions ■ Solve one-step equations having integer or decimal answers ■ Combine like terms (e.g., $2x + 5x$)	■ Locate points on the number line and in the first quadrant	■ Exhibit some knowledge of the angles associated with parallel lines	■ Compute the perimeter of polygons when all side lengths are given ■ Compute the area of rectangles when whole number dimensions are given
■ create expressions that model mathematical situations using combinations of symbols and numbers ■ evaluate algebraic expressions and solve multistep first-degree equations	■ sketch and identify line segments, midpoints, intersections, and vertical and horizontal lines	■ describe angles and triangles using mathematical terminology and apply their properties	■ find area and perimeter of a variety of polygons by substituting given values into standard geometric formulas
■ Evaluate algebraic expressions by substituting integers for unknown quantities ■ Add and subtract simple algebraic expressions ■ Solve routine first-degree equations ■ Perform straightforward word-to-symbol translations	■ Locate points in the coordinate plane	■ Find the measure of an angle using properties of parallel lines ■ Exhibit knowledge of basic angle properties and special sums of angle measures (e.g., 90°, 180°, and 360°)	■ Compute the area and perimeter of triangles and rectangles in simple problems ■ Use geometric formulas when all necessary information is given
■ identify, interpret, and generate symbolic representations that model the context of a problem ■ factor and perform the basic operations on polynomials ■ create and solve linear equations and inequalities that model real-world situations ■ solve literal equations for any variable	■ represent and interpret relationships defined by equations and formulas; translate between representations as ordered pairs, graphs, and equations; and investigate symmetry and transformations (e.g., reflections, translations, rotations)	■ recognize what geometric properties and relationships for parallel lines to apply to find unknown angle measures ■ recognize when to apply geometric properties and relationships of triangles to find unknown angle measures	■ apply a variety of strategies to determine the circumference or perimeter and the area for circles, triangles, rectangles, and composite geometric figures

EXPLORE MATHEMATICS TEST

Table 1 (continued): **The College Readiness Standards**

The Standards describe what students who score in the specified score ranges are *likely* to know and to be able to do. The ideas for progress help teachers identify ways of enhancing students' learning based on the scores students receive.

		Basic Operations & Applications	***Probability, Statistics, & Data Analysis***	***Numbers: Concepts & Properties***
24–25	**Standards**	■ Solve multistep arithmetic problems that involve planning or converting units of measure (e.g., feet per second to miles per hour)	■ Calculate the average, given the frequency counts of all the data values ■ Manipulate data from tables and graphs ■ Compute straightforward probabilities for common situations	■ Find and use the least common multiple ■ Order fractions ■ Work with numerical factors ■ Work with scientific notation ■ Work with squares and square roots of numbers
	ideas for progress	■ model and solve real-world problems that involve a combination of rates, proportions, and/or percents	■ find the probability of simple events, disjoint events, compound events, and independent events in a variety of settings using a variety of counting techniques	■ apply and use elementary number concepts and number properties to model and solve nonroutine problems that involve new ideas

Expressions, Equations, & Inequalities	*Graphical Representations*	*Properties of Plane Figures*	*Measurement*
■ Solve real-world problems using first-degree equations ■ Write expressions, equations, or inequalities with a single variable for common pre-algebra settings (e.g., rate and distance problems and problems that can be solved by using proportions) ■ Identify solutions to simple quadratic equations		■ Use several angle properties to find an unknown angle measure	■ Compute the area of triangles and rectangles when one or more additional simple steps are required ■ Compute the area and circumference of circles after identifying necessary information
■ create and use basic families of functions (which include linear, absolute value, and quadratic) to model and solve problems in common settings ■ explore and use different methods to solve systems of equations ■ manipulate radical expressions (e.g., rationalize denominators)	■ graph linear equations and inequalities, determine slopes of lines, identify parallel and perpendicular lines, and find distances ■ identify characteristics of figures from a general equation	■ apply special right-triangle properties and the Pythagorean theorem to solve congruent and similar shape problems	■ apply a variety of strategies using relationships between perimeter, area, and volume to calculate desired measures

Description of the EXPLORE Mathematics Test

What Does the EXPLORE Mathematics Test Measure?

The EXPLORE Mathematics Test is a 30-question, 30-minute test designed to assess the mathematical reasoning skills that students have typically acquired in courses taken up to the beginning of eighth grade. The multiple-choice test requires students to analyze problems in real-world and purely mathematical settings, plan and carry out solution strategies, and verify the appropriateness of solutions.

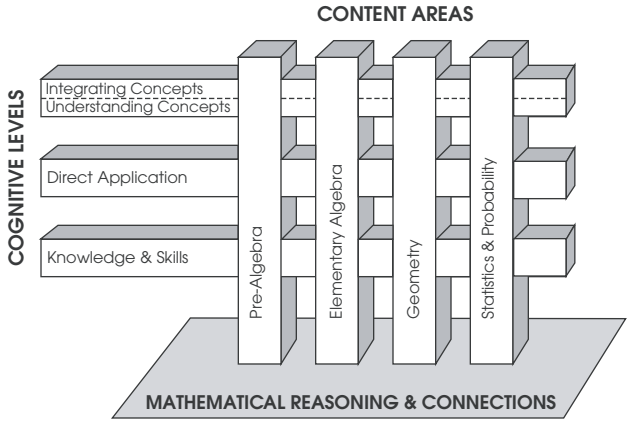
On the EXPLORE Mathematics Test, students demonstrate their ability to read and understand mathematical terminology; to apply definitions, algorithms, and properties; to interpret and analyze data; and to use mathematics to solve problems.

Students also apply quantitative reasoning in a variety of ways, such as discerning relationships between mathematical concepts, connecting and integrating mathematical concepts and ideas, and making generalizations. Computational skills and knowledge of basic formulas are assumed as background for the problems, but extensive computation and memorization of complex formulas are not required.

The questions focus on mathematical reasoning and making connections within and among four content areas and at various cognitive levels. These areas and levels are shown in Figure 1 below.

"The test should measure what students can do with what they have learned."

— (ACT, 1996b, p. 2)



Adapted from *Mathematics Framework for the 1996 National Assessment of Educational Progress* (p.11)

Figure 1: **EXPLORE Mathematics Test Content Areas and Cognitive Levels**

Through the various cognitive levels, students demonstrate their ability to use and reason with mathematics. *Knowledge and Skills* questions (about 27% of the test) require students to use one or more facts, definitions, formulas, or procedures to solve problems that are presented in purely mathematical terms. *Direct Application* questions (about 27% of the test) require students to use their knowledge and skills to solve straightforward problems set in real-world situations. *Understanding Concepts* and *Integrating Conceptual Understanding* questions (about 46% of the test) assess students' depth of understanding of major concepts by requiring reasoning from a single concept or the integration of several concepts to reach an inference or a conclusion.

The content of the EXPLORE Mathematics Test is reflective of the content taught in mathematics classrooms and of the prerequisite skills and understandings necessary for high school mathematics courses. ACT routinely monitors the secondary school mathematics curriculum through reviews of state and national standards, current textbooks, and national organizations' curriculum frameworks; surveys of secondary and postsecondary instructors; and meetings with education consultants. A brief description of the content sampled on the test and the approximate percentage of the test devoted to each content area on the EXPLORE Mathematics Test are provided below.

Pre-Algebra (33%). Questions in this content area are based on basic operations using whole numbers, decimals, fractions, and integers. Topics include place value, square roots, scientific notation, factors, ratio and proportion, and percent.

Elementary Algebra (30%). Questions in this content area are based on operations with algebraic expressions, including evaluation of algebraic expressions by substitution, use of variables to express functional relationships, solution of linear equations in one variable, use of real number lines to represent numbers, and graphing points in the standard coordinate plane.

Geometry (23%). Questions in this content area are based on use of scales and measurement systems, plane and solid geometric figures and associated relationships and concepts, the concept of angles and their measures, parallelism, relationships of triangles, properties of a circle, and the Pythagorean theorem. All of these topics are addressed at a level preceding formal geometry.

Statistics/Probability (14%). Questions in this content area are based on elementary counting and rudimentary probability; data collection, representation, and interpretation; and reading and relating graphs, charts, and other representations of data. These topics are addressed at a level preceding formal statistics.

What Is ACT's Calculator Policy for EXPLORE?

Students are expected to have calculators available when taking the EXPLORE Mathematics Test and are encouraged to use the calculator they are most comfortable with. Many questions on the test may be solved with or without a calculator, neither strategy being clearly superior to the other. The test also includes problems for which a calculator is clearly the best tool to use, and others where a non-calculator solution is recommended. Students must choose when to use and when not to use calculators.

How Are the Test Questions Linked to the College Readiness Standards?

The EXPLORE Mathematics Test assesses various kinds and combinations of skills; each of these skills can be measured in different ways. You may have noticed that the strands and the content areas are not the same. The strands are areas in which there are a variety of test questions representing a continuum of skills and understandings. The strands are similar to those found in state and national frameworks. Many of the strands cut across the different content areas on the EXPLORE Mathematics Test.

Table 2 provides the strands and the corresponding content areas.

Table 2: **EXPLORE Mathematics Strands and Corresponding Content Areas**

Strand	**Content Area**
Basic Operations & Applications	Pre-Algebra
Probability, Statistics, & Data Analysis	Statistics/Probability
Numbers: Concepts & Properties	Pre-Algebra
Expressions, Equations, & Inequalities	Pre-Algebra Elementary Algebra
Graphical Representations	Elementary Algebra Geometry
Properties of Plane Figures	Geometry
Measurement	Geometry

The Need for Thinking Skills

Every student comes to school with the ability to think, but to achieve their goals students need to develop skills such as learning to make new connections between texts and ideas, to understand increasingly complex concepts, and to think through their assumptions. Because of technological advances and the fast pace of our society, it is increasingly important that students not only know information but also know how to critique and manage that information. Students must be provided with the tools for ongoing learning; understanding, analysis, and generalization skills must be developed so that the learner is able to adapt to a variety of situations.

How Are EXPLORE Test Questions Linked to Thinking Skills?

Our belief in the importance of developing thinking skills in learners was a key factor in the development of EXPLORE. ACT believes that students' preparation for further learning is best assessed by measuring, as directly as possible, the academic skills that students have acquired and that they will need to perform at the next level of learning. The required academic skills can most directly be assessed by reproducing as faithfully as possible the complexity of the students' schoolwork. Therefore, the EXPLORE test questions are designed to determine how skillfully students solve problems, grasp implied meanings, draw inferences, evaluate ideas, and make judgments in subject-matter areas important to success in intellectual work both inside and outside school.

Table 3 on pages 19–25 provides sample test questions, organized by score range, that are linked to specific skills within each of the seven Mathematical strands. It is important to note the increasing level of skill with mathematics—computing, reasoning, and making connections—that students scoring in the higher score ranges are able to demonstrate. The questions were chosen to illustrate the variety of content as well as the range of complexity within each strand. The sample test questions for the 13–15, 16–19, 20–23, and 24–25 score ranges are the kinds of items answered correctly by 80% or more of the EXPLORE examinees who obtained scores in each of these four score ranges.

As you review the sample test questions, you will note that each correct answer is marked with an asterisk. For score ranges that include more than one skill, boldface type is used to denote the skill that best corresponds to the sample test question.

"Learning is not attained by chance, it must be sought for with ardour and attended to with diligence."

— Abigail Adams in a letter to John Quincy Adams

Table 3: **EXPLORE Sample Test Questions by Score Range**
Basic Operations & Applications Strand

Score Range	*Basic Operations & Applications*	*Sample Test Questions*
13–15	Perform one-operation computation with whole numbers and decimals **Solve problems in one or two steps using whole numbers** Perform common conversions (e.g., inches to feet or hours to minutes)	Central High's musical event must make $780 in order to break even. If each ticket costs $6, how many tickets must be sold to break even? **A.** 125 ***B.** 130 **C.** 138 **D.** 180 **E.** 774
16–19	**Solve routine one-step arithmetic problems (using whole numbers, fractions, and decimals) such as single-step percent** Solve some routine two-step arithmetic problems	What is 12% of 60 ? **A.** 6 ***B.** 7.2 **C.** 12 **D.** 48 **E.** 72
20–23	Solve routine two-step or three-step arithmetic problems involving concepts such as rate and proportion, tax added, percentage off, and computing with a given average	A student on the local softball team has batted 40 times and has 24 hits. At this rate, how many hits will she have if she bats 100 times? **A.** 48 ***B.** 60 **C.** 68 **D.** 72 **E.** 84
24–25	Solve multistep arithmetic problems that involve planning or converting units of measure (e.g., feet per second to miles per hour)	Sally and Peter have a lawn-mowing business. They earn $30 for a large yard, $21 for a medium-sized yard, and $15 for a small yard. If they mow 8 yards and there are at least 2 yards of each size, what is the maximum amount of money they can earn? **A.** $ 66 **B.** $132 **C.** $176 ***D.** $192 **E.** $240

Table 3: **EXPLORE Sample Test Questions by Score Range**
Probability, Statistics, & Data Analysis Strand

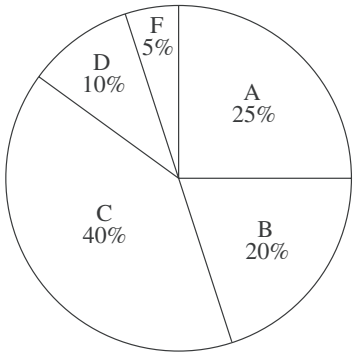
Score Range	Probability, Statistics, & Data Analysis	Sample Test Questions
13–15	Calculate the average of a list of positive whole numbers Perform a single computation using information from a table or chart	Due to the secure nature of the test, it was not possible to provide a sample test question for these skills.
16–19	Calculate the average of a list of numbers Calculate the average, given the number of data values and the sum of the data values Read tables and graphs **Perform computations on data from tables and graphs** Use the relationship between the probability of an event and the probability of its complement	A survey taken at a local preschool asked each child for his or her favorite number. The following table shows the results. How many of the children's favorite numbers were also their age? Favorite Number Age | 2 | 3 | 4 | 5 | Over 5 3 | 4 | 8 | 7 | 2 | 1 4 | 2 | 9 | 9 | 8 | 3 5 | 2 | 4 | 1 | 12 | 5 **A.** 14 **B.** 20 ***C.** 29 **D.** 34 **E.** 77
20–23	Calculate the missing data value, given the average and all data values but one Translate from one representation of data to another (e.g., a bar graph to a circle graph) **Determine the probability of a simple event**	A box contains 8 marbles: 1 solid red, 3 blue speckled, 2 green speckled, and 2 solid yellow. If Omar picks a marble at random from the box, what is the probability that it will be speckled? **A.** $\frac{1}{8}$ **B.** $\frac{1}{4}$ **C.** $\frac{3}{8}$ **D.** $\frac{3}{5}$ ***E.** $\frac{5}{8}$
24–25	Calculate the average, given the frequency counts of all the data values **Manipulate data from tables and graphs** Compute straightforward probabilities for common situations	The following graph shows what percentage of the 180 students in social studies received each of the possible grades (A–F). How many of the students in social studies received an A ? **A.** 25 **B.** 35 ***C.** 45 **D.** 55 **E.** 65 

Table 3: **EXPLORE Sample Test Questions by Score Range**
Numbers: Concepts & Properties Strand

Score Range	***Numbers: Concepts & Properties***	***Sample Test Questions***
13–15	Recognize equivalent fractions and fractions in lowest terms	The fraction $\frac{16}{36}$ is the same as: **A.** $\frac{17}{37}$ **B.** $\frac{1}{21}$ ***C.** $\frac{4}{9}$ **D.** $\frac{2}{3}$ **E.** $\frac{1}{3}$
16–19	**Recognize one-digit factors of a number** Identify a digit's place value	What is the greatest whole number that is a factor of both 36 and 40 ? ***A.** 4 **B.** 5 **C.** 6 **D.** 8 **E.** 12
20–23	Exhibit knowledge of elementary number concepts including rounding, the ordering of decimals, pattern identification, absolute value, primes, and greatest common factor	Which of the following numbers is the greatest? **A.** $0.\overline{2324}$ **B.** $0.2\overline{324}$ **C.** $0.23\overline{24}$ ***D.** $0.232\overline{4}$ **E.** 0.2324
24–25	Find and use the least common multiple Order fractions Work with numerical factors Work with scientific notation Work with squares and square roots of numbers	Due to the secure nature of the test, it was not possible to provide a sample test question for these skills.

Table 3: **EXPLORE Sample Test Questions by Score Range**
Expressions, Equations, & Inequalities Strand

Score Range	***Expressions, Equations, & Inequalities***	***Sample Test Questions***
13–15	**Exhibit knowledge of basic expressions (e.g., identify an expression for a total as *b* + *g*)** Solve equations in the form $x + a = b$, where *a* and *b* are whole numbers or decimals	If *f* is the number of first graders who play soccer and *s* is the number of second graders who play soccer, which of the following expressions represents the total number of first and second graders who play soccer? ***A.** $f + s$ **B.** $f - s$ **C.** $s - f$ **D.** $f \times s$ **E.** $f \div s$
16–19	Substitute whole numbers for unknown quantities to evaluate expressions **Solve one-step equations having integer or decimal answers** Combine like terms (e.g., $2x + 5x$)	If $x + 14.520 = 105.149$, then x = ? ***A.** 90.629 **B.** 103.697 **C.** 106.601 **D.** 119.669 **E.** 250.349
20–23	Evaluate algebraic expressions by substituting integers for unknown quantities Add and subtract simple algebraic expressions Solve routine first-degree equations **Perform straightforward word-to-symbol translations**	Seven people made a total of 133 phone calls. What equation would you use to calculate the average number (*N*) of phone calls per person? ***A.** $7N = 133$ **B.** $133N = 7$ **C.** $7 \times 133 = N$ **D.** $N \times 133 = 7$ **E.** $133 + 7 = N$
24–25	Solve real-world problems using first-degree equations Write expressions, equations, or inequalities with a single variable for common pre-algebra settings (e.g., rate and distance problems and problems that can be solved by using proportions) **Identify solutions to simple quadratic equations**	If $x = 0.7$, which of the following values of *y* makes the equation below true? $5x + y^2 = 9.75$ ***A.** 2.5 **B.** 3.125 **C.** 3.14 **D.** 4.05 **E.** 6.25

Table 3: **EXPLORE Sample Test Questions by Score Range**
Graphical Representations Strand

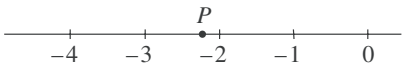
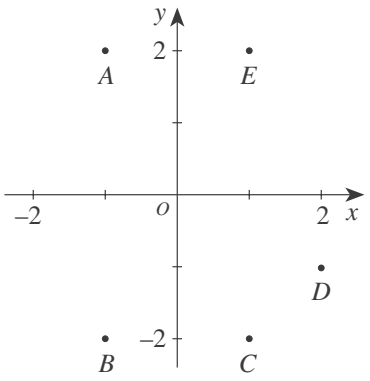
Score Range	Graphical Representations	Sample Test Questions
13–15	Identify the location of a point with a positive coordinate on the number line	Due to the secure nature of the test, it was not possible to provide a sample test question for this skill.
16–19	Locate points on the number line and in the first quadrant	Which of the following is closest to the coordinate of point P shown on the real number line below?  **A.** $-3\frac{3}{4}$ **B.** $-3\frac{1}{4}$ **C.** $-2\frac{3}{4}$ ***D.** $-2\frac{1}{4}$ **E.** -2
20–23	Locate points in the coordinate plane	One of the points, labeled A through E, shown in the standard (x,y) coordinate plane below has coordinates $(-1,2)$. Which point is it? ***A.** A **B.** B **C.** C **D.** D **E.** E 
24–25		

Table 3: **EXPLORE Sample Test Questions by Score Range**
Properties of Plane Figures Strand

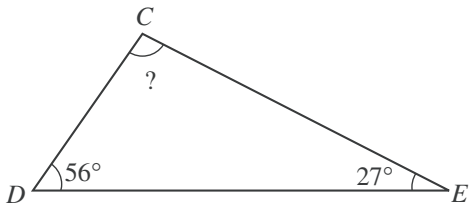
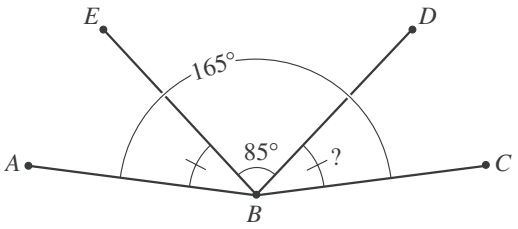
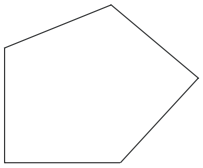
Score Range	Properties of Plane Figures	Sample Test Questions
13–15		
16–19	Exhibit some knowledge of the angles associated with parallel lines	Due to the secure nature of the test, it was not possible to provide a sample test question for this skill.
20–23	Find the measure of an angle using properties of parallel lines **Exhibit knowledge of basic angle properties and special sums of angle measures (e.g., 90°, 180°, and 360°)**	What is the measure of $\angle C$ in $\triangle CDE$ below? **A.** 83° ***B.** 97° **C.** 103° **D.** 107° **E.** 113° 
24–25	Use several angle properties to find an unknown angle measure	In the figure below, the measure of $\angle ABD$ is equal to the measure of $\angle EBC$. Also, the measure of $\angle ABC$ is 165° and the measure of $\angle DBE$ is 85°. What is the measure of $\angle DBC$? ***A.** 40° **B.** $42\frac{1}{2}°$ **C.** 45° **D.** 50° **E.** 55° 

Table 3: **EXPLORE Sample Test Questions by Score Range**
Measurement Strand

Score Range	Measurement	Sample Test Questions
13–15	Estimate or calculate the length of a line segment based on other lengths given on a geometric figure	Due to the secure nature of the test, it was not possible to provide a sample test question for this skill.
16–19	**Compute the perimeter of polygons when all side lengths are given** Compute the area of rectangles when whole number dimensions are given	The length of each side of the pentagon shown below is 17 millimeters. What is the perimeter of the pentagon in millimeters? **A.** 22 **B.** 44 **C.** 68 ***D.** 85 **E.** 102 
20–23	Compute the area and perimeter of triangles and rectangles in simple problems **Use geometric formulas when all necessary information is given**	The formula $V = \frac{1}{3}\pi r^2 h$ is used to find the volume (V) of a right circular cone with a radius (r), height (h), and $\pi \approx 3.14$. To the nearest cubic inch, what is the volume of a cone with a height of 10 inches and a radius of 3 inches? **A.** 31 ***B.** 94 **C.** 188 **D.** 283 **E.** 314
24–25	Compute the area of triangles and rectangles when one or more additional simple steps are required **Compute the area and circumference of circles after identifying necessary information**	A circular table top has a radius of 3.5 feet. What is the area of the table top, rounded to the nearest whole number of square feet? **A.** 11 **B.** 22 **C.** 33 ***D.** 38 **E.** 121

Thinking Your Way Through the EXPLORE Test

In our increasingly complex society, students' ability to think critically and make informed decisions is more important than ever. The workplace demands new skills and knowledge and continual learning; information bombards consumers through media and the Internet; familiar assumptions and values often come into question. More than ever before, students in today's classrooms face a future when they will need to adapt quickly to change, to think about issues in rational and creative ways, to cope with ambiguities, and to find means of applying information to new situations.

Classroom teachers are integrally involved in preparing today's students for their futures. Such preparation must include the development of thinking skills such as problem solving, decision making, and inferential and evaluative thinking. These are, in fact, the types of skills and understandings that underlie the test questions on EXPLORE.

How Can Analyzing Test Questions Build Thinking Skills?

On pages 27–28, you will find additional sample test questions. The sample test questions provide a link to a strand, a Standard, and a score range. Each sample test question includes a description of the skills and understandings students must demonstrate in order to arrive at the correct answer. The descriptions provide a series of strategies students typically might employ as they work through each test question. Possible flawed strategies leading to the choice of one or more incorrect responses also are offered. Analyzing test questions in this way, as test developers do to produce a Test Question Rationale, can provide students with a means of understanding the knowledge and skills embedded in the test questions and an opportunity to explore why an answer choice is correct or incorrect.

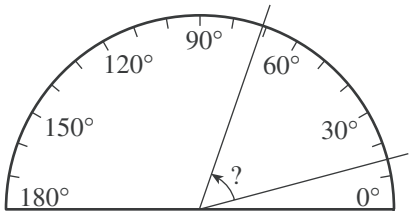
Providing students with strategies such as these encourages them to take charge of their thinking and learning. The sample test questions that appear in Table 3 on pages 19–25 can be used to develop additional Test Question Rationales.

"Learning is fundamentally about *making and maintaining connections* . . . among concepts, ideas, and meanings."

— American Association for Higher Education, American College Personnel Association, & National Association of Student Personnel Administrators, June 1998

Test Question Rationale	
Properties of Plane Figures	■ Exhibit knowledge of basic angle properties and special sums of angle measures (e.g., 90°, 180°, and 360°) ■ 20–23 score range

1. Shown below is a protractor in position to measure an angle. Which of the following is closest to the measure of that angle?



A. 15°
B. 45°
***C.** 57°
D. 72°
E. 87°

Question 1 addresses several basic concepts of measurement, including understanding the concept of degree in angles, knowledge of how to use a protractor and how to read a scale, and the ability to adapt thinking since neither of the rays that form the angle is at the 0° mark.

In order to find the measure of the angle in question, a student might choose one of several strategies. For example, some students might imagine rotating the protractor counterclockwise until the close side of the angle goes through the 0° mark, a rotation of about 15°. That would make the second side go through the (72°–15°) mark and the angle would measure about 57°—answer C. Other students would more naturally subtract the measure of the angle from 0° to 15° from the measure of the angle from 0° to 72°, rather than imagining rotating the protractor into standard measuring position. While both strategies would yield the correct answer (C), the methods are subtly different.

The most common incorrect answer is 72° (D), which is chosen when students fail to account for the angle not being lined up with the 0° mark. Students who choose answer A could be ignoring the arrow indicating which angle is being measured. Students choosing answer B could simply be misreading the protractor, or they may be looking only at the angle and seeing that it resembles a 45° angle. Answer choice E might be eliminated based on the student's knowledge of what a 90° angle looks like. Students who have had experience using protractors should be able to correctly answer this question.

This question is classified KS (Knowledge and Skills) because it is in a purely mathematical context and students may be used to lining up rulers away from the end in order to get a more accurate measurement.

Test Question Rationale	
Probability, Statistics, & Data Analysis	■ Perform computations on data from tables and graphs ■ 16–19 score range

2. A survey taken at a local preschool asked each child for his or her favorite number. The following chart shows the results. How many of the children's favorite numbers were also their age?

	Favorite Number				
Age	2	3	4	5	Over 5
3	4	8	7	2	1
4	2	9	9	8	3
5	2	4	1	12	5

F. 14
G. 20
*__H.__ 29
J. 34
K. 77

Question 2 involves reading and interpreting data presented in a chart. Students must be able to analyze the chart, understand how the rows and columns are labeled, and draw a conclusion based on the evidence presented in the chart. The correct answer cannot be found by simply locating one number on the chart; rather, students have to take the statement about the children's favorite numbers being their age and connect that statement to the row and column headings of the chart. Students who select the correct answer will determine that there are 8 children who are 3 years old whose favorite number is 3; there are 9 children who are 4 years old whose favorite number is 4; and there are 12 children who are 5 years old whose favorite number is 5. This makes 8 + 9 + 12 = 29 children whose favorite number is their age (answer H). Some students might figure out that the numbers involved (8, 9, and 12) appear on a diagonal line on the chart. Answers F (14) and G (20) might be selected by students who added 4 + 9 + 1 or 7 + 8 + 5 on other diagonals. Students who select answer choice K might do so because of a lack of clear understanding of the problem's question, and/or because they added all the numbers on the chart.

This question is classified DA (Direct Application) because it includes the real-world context of a survey conducted at a preschool.

The Assessment-Instruction Link

Why Is It Important to Link Assessment With Instruction?

Assessment provides feedback to the learner and the teacher. It bridges the gap between expectations and reality. Assessment can gauge the learners' readiness to extend their knowledge in a given area, measure knowledge gains, identify needs, and determine the learners' ability to transfer what was learned to a new setting.

When teachers use assessment tools to gather information about their students, then modify instruction accordingly, the assessment process becomes an integral part of teaching and learning. Using assessment to inform instruction can help teachers create a successful learning environment.

Students can use assessment as a tool to help them revise and rethink their work, to help integrate prior knowledge with new learning, and to apply their knowledge to new situations. Connecting assessment to classroom instruction can help both teachers and students take charge of thinking and learning.

"*Every* objective, *every* lesson plan, *every* classroom activity, and *every* assessment method should focus on helping students achieve those [significant] outcomes that will help students both in the classroom and beyond."

— Kay Burke, editor of *Authentic Assessment: A Collection*

As teachers review student performances on various measures, they can reexamine how to help students learn. As Peter Airasian, the author of *Classroom Assessment,* says, "Assessment is not an end in itself, but a means to another end, namely, good decision making" (p. 19). Linking assessment and instruction prompts both teachers and students to take on new roles and responsibilities. Through reflecting together on their learning, students and teachers can reevaluate their goals and embark on a process of continuous growth.

Are Your Students Developing the Necessary Skills?

EXPLORE can be administered in eighth or ninth grade to provide students with an early indication of their educational progress in the context of the post-high school educational and career options they are considering. The results from EXPLORE can be used to help students make adjustments in their course work to help ensure that they are prepared for what they want to do in and after high school.

EXPLORE and PLAN are developmentally and conceptually linked to the ACT and thus provide a coherent framework for students and counselors and a consistent skills focus for teachers from Grades 8 through 12.

Because EXPLORE is linked to PLAN, students receive an estimated PLAN Composite score along with their EXPLORE scores. These scores can be used to evaluate students' readiness for high school and to plan an appropriate course of study.

As students and others review test scores from EXPLORE, PLAN, and the ACT, they should be aware that ACT's data clearly reveal that students' ACT test scores are directly related to preparation for college. Students who take rigorous high school courses, which ACT has defined as core college preparatory courses, achieve much higher test scores than students who do not. ACT has defined core college preparatory course work as four or more years of English, and three or more years each of mathematics, social studies, and natural science.

ACT works with colleges to help them develop guidelines that place students in courses that are appropriate for their level of achievement as measured by the ACT. In doing this work, ACT has gathered course grade and test score data from a large number of first-year students across a wide range of postsecondary institutions. These data provide an overall measure of what it takes to be successful in a standard first-year college course. Data from 98 institutions and over 90,000 students were used to establish the ACT College Readiness Benchmark Scores, which are median course placement scores achieved on the ACT that are directly reflective of student success in a college course.

Success is defined as a 50 percent chance that a student will earn a grade of B or better. The courses are the ones most commonly taken by first-year students in the areas of English, mathematics, social studies, and science, namely English Composition, College Algebra, an entry-level College Social Studies/Humanities course, and College Biology. The ACT scores established as the ACT College Readiness Benchmark Scores are 18 on the English Test, 22 on the Mathematics Test, 21 on the Reading Test, and 24 on the Science Test. The College Readiness Benchmark Scores were based upon a sample of postsecondary institutions from across the United States. The data from these institutions were weighted to reflect postsecondary institutions nationally. The Benchmark Scores are median course placement values for these institutions and as such represent a *typical* set of expectations.

College Readiness Benchmark Scores have also been developed for EXPLORE and for PLAN, to indicate a student's probable readiness for college-level work, in the same courses named above, by the time the student graduates from high school. The EXPLORE and PLAN College Readiness Benchmark Scores were developed using records of students who had taken EXPLORE, PLAN, and the ACT (four years of matched data). Using either EXPLORE subject-area scores or PLAN subject-area scores, we estimated the conditional probabilities associated with meeting or exceeding the corresponding ACT Benchmark Score. Thus, each EXPLORE (1–25) or PLAN (1–32) score was associated with an estimated probability of meeting or exceeding the relevant ACT Benchmark Score. We then identified the EXPLORE and PLAN scores, at Grades 8, 9, 10, and 11, that came the closest to a 0.5 probability of meeting or exceeding the ACT Benchmark Score, by subject area. These scores were selected as the EXPLORE and PLAN Benchmark Scores.

All the Benchmark Scores are given in Table 4. Note that, for example, the first row of the table should be read as follows: An eighth-grade student who scores 13, or a ninth-grade student who scores 14, on the EXPLORE English Test has a 50 percent probability of scoring 18 on the ACT English Test; and a tenth-grade student who scores 15, or an eleventh-grade student who scores 17, on the PLAN English Test has a 50 percent probability of scoring 18 on the ACT English Test.

Table 4: **College Readiness Benchmark Scores**

	EXPLORE Test Score		PLAN Test Score		ACT
Subject Test	**Grade 8**	**Grade 9**	**Grade 10**	**Grade 11**	**Test Score**
English	13	14	15	17	18
Mathematics	17	18	19	21	22
Reading	15	16	17	19	21
Science	20	20	21	23	24

Using Assessment Information to Help Support Low-Scoring Students

Students who receive a Composite score of 13 or below on EXPLORE will most likely require additional guidance and support from their teachers and family in order to meet their academic goals, particularly if one of those goals is to attend a four-year college or university.

Because EXPLORE, PLAN, and the ACT share a common score scale, each student who takes EXPLORE receives an estimated PLAN Composite score range. This estimated score range predicts how a student might expect to perform on PLAN as a high school sophomore. The estimated score ranges, for both eighth-grade test takers and ninth-grade test takers, are reported in Table 5.

Table 5 indicates that, for an EXPLORE Composite score of 12 when EXPLORE is taken in Grade 8, the lower limit of the estimated PLAN Composite score range is given as 13 and the upper limit is given as 16. That is, an estimated PLAN Composite score range of 13 to 16 is reported for eighth-grade students with EXPLORE Composite scores of 12. Similarly, when EXPLORE is taken in Grade 9, a student's EXPLORE Composite score of 12 results in an estimated PLAN Composite score range of 11 to 14.

Since both EXPLORE and PLAN are designed to be curriculum-based testing programs, some students' performance on PLAN will fall outside their estimated PLAN Composite score range. If students do not maintain good academic work in school, their actual PLAN Composite scores may fall short of their estimated score ranges. Conversely, some students who improve their academic performance may earn PLAN Composite scores higher than their estimated score ranges.

Eighth or ninth grade is a good time for students, parents, counselors, and teachers to take stock of a student's progress. EXPLORE test scores and other performance indicators should be discussed in the context of the student's future goals, previous academic preparation, and plans for future high school course work.

Table 5: **Estimated PLAN Composite Score Ranges**

EXPLORE Composite Score	Estimated PLAN Composite Score Range for 8th Graders		for 9th Graders	
	Low	High	Low	High
1	8	11	8	12
2	8	11	8	12
3	8	11	8	12
4	8	11	8	12
5	10	13	8	12
6	10	13	9	12
7	10	13	9	12
8	10	13	9	12
9	10	13	9	12
10	11	14	10	13
11	12	15	11	14
12	13	16	11	14
13	14	17	12	15
14	15	18	13	16
15	16	19	14	17
16	17	20	15	18
17	18	21	16	19
18	19	23	18	21
19	19	23	19	22
20	20	24	20	24
21	21	25	21	25
22	23	27	22	26
23	24	28	23	27
24	25	29	24	28
25	27	30	26	30

As educators and parents look over a student's academic performance, the way the student's scores and goals match up can suggest a course of action. For example, a student who wishes to become an engineer will need a solid mathematics background. A high Mathematics Test score can be used as evidence that the goal is realistic. A low score (or subscore) suggests the student should consider ways of improving his or her mathematics skills through additional course work and/or additional assistance in the area.

First, using the College Readiness Standards, school personnel might explain EXPLORE scores to students and parents. Then, using reports and test data from classroom teachers, grade point averages, and data from district and state tests, educators and parents can help students make decisions about which academic areas students might need additional assistance with, which student goals might need to be redirected, and which junior high or high school courses to take.

Eighth or ninth grade is a good time to begin taking demanding course work (Noeth & Wimberly, 2002). "Many studies have found . . . that taking high-level math courses increases college-going among minority and first generation college students. Further, students who take algebra in eighth grade are very likely to apply to a four-year college, controlling for other high school course taking" (Noeth & Wimberly, 2002, p. 17).

In addition to planning for high school course work, taking remedial classes if necessary, and beginning to match career goals to known talents, eighth-grade students who want to attend a four-year college or university should begin educating themselves about such schools. Some students, particularly those whose parents did not attend college, may not have access to information about postsecondary education. "Though many students . . . attending urban schools may have the desire and expectation, they may not have the skills, knowledge, and information they need to enter and complete a postsecondary program. Many . . . do not have the informational resources, personal support networks, continual checkpoints, or structured programs to make college exploration and planning a theme throughout their daily lives. . . . Students need their schools, parents, and others to help them plan for college and their future careers" (Noeth & Wimberly, 2002, p. 4).

College admission policies vary widely in their level of selectivity. ACT Composite scores typically required by colleges having varying levels of selectivity are shown in Table 6. This information provides only general guidelines. There is considerable overlap among admission categories, and colleges often make exceptions to their stated admission policies.

Table 6: **The Link Between ACT Composite Scores and College Admission Policies**

Admission Policy	Typical Class Rank of Admitted Students	Typical ACT Composite Scores of Admitted Students
Highly Selective	Majority of accepted freshmen in top 10% of high school graduating class	25–30
Selective	Majority of accepted freshmen in top 25% of high school graduating class	21–26
Traditional	Majority of accepted freshmen in top 50% of high school graduating class	18–24
Liberal	Some of accepted freshmen from lower half of high school graduating class	17–22
Open	All high school graduates accepted to limit of capacity	16–21

What Does It Mean to Be a Low-Scoring Student?

Low-achieving students tend to be those students who score low on standardized tests. Students who slip behind are the likeliest to drop out of school and least likely to overcome social and personal disadvantages.

According to Judson Hixson, a researcher at the North Central Regional Educational Laboratory (NCREL), students who are at risk should be considered in a new light:

> Students are placed "at risk" when they experience a significant mismatch between their circumstances and needs, and the capacity or willingness of the school to accept, accommodate, and respond to them in a manner that supports and enables their maximum social, emotional, and intellectual growth and development.
>
> As the degree of mismatch increases, so does the likelihood that they will fail to either complete their elementary and secondary education, or more importantly, to benefit from it in a manner that ensures they have the knowledge, skills, and dispositions necessary to be successful in the next stage of their lives—that is, to successfully pursue post-secondary education, training, or meaningful employment and to participate in, and contribute to, the social, economic, and political life of their community and society as a whole.
>
> The focus of our efforts, therefore, should be on enhancing our institutional and professional capacity and responsiveness, rather than categorizing and penalizing students for simply being who they are. (Hixson, 1993, p. 2)

Hixson's views reveal the necessity of looking at all the variables that could affect students' performance, not just focusing on the students themselves.

Low-achieving students may demonstrate some of the following characteristics:

- difficulty with the volume of work to be completed;
- low reading and writing skills;
- low motivation;
- low self-esteem;
- poor study habits;
- lack of concentration;
- reluctance to participate in class or to ask for help with tasks/assignments; and
- test anxiety.

Many of these characteristics are interconnected. For example, a low-scoring student cannot complete the volume of work a successful student can if it takes a much longer time for that low-scoring student to decipher text passages because of low reading skills. There is also the issue of intrinsic motivation: students may have little desire to keep trying if they do not habitually experience success.

Some low-scoring students may not lack motivation or good study habits, but may still be in the process of learning English; still others may have learning disabilities that make it difficult for them to do complex work in one or two content areas.

Again, we must not focus only on the students themselves, but also consider other variables that could affect their academic performance, such as

- job or home responsibilities that take time away from school responsibilities;
- parental attitude toward and involvement in students' school success;
- students' relationships with their peers;
- lack of adequate support and resources; and
- lack of opportunities.

For example, some students who score low on tests are never introduced to a curriculum that challenges them or that addresses their particular needs: "Much of the student stratification within academic courses reflects the social and economic stratification of society. Schools using tracking systems or other methods that ultimately place low-income and marginal students in lower-level academic courses are not adequately preparing them to plan for postsecondary education, succeed in college, and prepare for lifelong learning" (Noeth & Wimberly, 2002, p. 18).

As Barbara Means and Michael Knapp have suggested, many schools need to reconstruct their curricula, employing instructional strategies that help students to understand how experts think through problems or tasks, to discover multiple ways to solve a problem, to complete complex tasks by receiving support (e.g., cues, modifications), and to engage actively in classroom discussions (1991).

Many individuals and organizations are interested in helping students succeed in the classroom and in the future. For example, the Network for Equity in Student Achievement (NESA), a group of large urban school systems, and the Minority Student Achievement Network (MSAN), a group of school districts in diverse suburban areas and small cities, are organizations that are dedicated to initiating strategies that will close the achievement gap among groups of students. Many schools and districts have found participation in such consortia to be helpful.

According to Michael Sadowski, editor of the *Harvard Education Letter,* administrators and teachers who are frustrated by persistent achievement gaps within their school districts "have started to look for answers within the walls of their own schools. They're studying school records, disaggregating test score and grade data, interviewing students and teachers, administrating questionnaires—essentially becoming researchers—to identify exactly where problems exist and to design solutions" (Sadowski, 2001, p. 1).

A student may get a low score on a standardized test for any of a number of reasons. To reduce the probability of that outcome, the following pages provide information about factors that affect student performance as well as some suggestions about what educators and students can do before students' achievement is assessed on standardized tests like EXPLORE.

What Are Some Factors That Affect Student Performance?

Many factors affect student achievement. Diane Ravitch, a research professor at New York University, has identified several positive factors in her book *The Schools We Deserve: Reflections on the Educational Crisis of Our Time* (1985, pp. 276 and 294). These factors, which were common to those schools that were considered effective in teaching students, include

- a principal who has a clearly articulated vision for the school, and the leadership skills to empower teachers to work toward that vision;
- a strong, clearly thought-out curriculum in which knowledge gained in one grade is built upon in the next;
- dedicated educators working in their field of expertise;
- school-wide commitment to learning, to becoming a "community of learners";
- a blend of students from diverse backgrounds;
- "high expectations for all" students; and
- systematic monitoring of student progress through an assessment system.

There are also factors that have a negative impact on student achievement. For example, some students "may not know about, know how, or feel entitled to take academic advantage of certain opportunities, like college preparatory courses, college entrance exams, and extracurricular learning opportunities" (Goodwin, 2000, p. 3).

All students need to be motivated to perform well academically, and they need informed guidance in sorting out their educational/career aspirations. Teachers who challenge their students by providing a curriculum that is rigorous and relevant to their world and needs (Brewer, Rees, & Argys, 1995; Gay, 2000), and who have a degree and certification in the area in which they teach (Ingersoll, 1998) and ample opportunities to collaborate with their peers (McCollum, 2000), are more likely to engender students' success in school.

Making the Invisible Visible

Using assessment information, such as that provided by the EXPLORE, PLAN, and ACT tests in ACT's Educational Planning and Assessment System (EPAS), can help bring into view factors that may affect—either positively or negatively—student performance. Reviewing and interpreting assessment information can encourage conversations between parents and teachers about what is best for students. Using data is one way of making the assumptions you have about your students and school, or the needs of students, visible.

Collecting assessment information in a systematic way can help teachers in various ways. It can help teachers see more clearly what is happening in their classrooms, provide evidence that the method of teaching they're using really works, and determine what is most important to do next. As teachers become active teacher-researchers, they can gain a sense of control and efficacy that contributes to their sense of accomplishment about what they do each day.

There are many different types of assessment information that a school or school district can collect. Some types yield quantitative data (performance described in numerical terms), others qualitative data (performance described in nonnumerical terms, such as text, audio, video, or photographs). All types, when properly analyzed, can yield useful insights into student learning. For example, schools and teachers can collect information from

- standardized tests (norm- or criterion-referenced tests);
- performance assessments (such as portfolios, projects, artifacts, presentations);
- peer assessments;
- progress reports (qualitative, quantitative, or both) on student skills and outcomes;
- self-reports, logs, journals; and
- rubrics and rating scales.

Reviewing student learning information in the context of demographic data may also provide insight and information about specific groups of students, like low-scoring students. Schools therefore would benefit by collecting data about

- enrollment, mobility, and housing trends;
- staff and student attendance rates and tardiness rates;
- dropout, retention, and graduation rates;
- gender, race, ethnicity, and health;
- percent of free/reduced lunch and/or public assistance;
- level of language proficiency;
- staff/student ratios;
- number of courses taught by teachers outside their endorsed content area;
- retirement projections and turnover rates; and
- teaching and student awards.

What Can Educators and Students Do Before Students Take Standardized Tests?

Integrate assessment and instruction. Because EXPLORE is curriculum-based, the most important prerequisite for optimum performance on the test is a sound, comprehensive educational program. This "preparation" begins long before any test date. Judith Langer, the director of the National Research Center on English Learning and Achievement, conducted a five-year study that compared the English programs of typical schools to those that get outstanding results. Schools with economically disadvantaged and diverse student populations in California, Florida, New York, and Texas predominated the study. Langer's study revealed that in higher performing schools "test preparation has been integrated into the class time, as part of the ongoing English language arts learning goals." This means that teachers discuss the demands of high-stakes tests and how they "relate to district and state standards and expectations as well as to their curriculum" (Langer, 2000, p. 6).

Emphasize core courses. ACT research conducted in urban schools both in 1998 and 1999 shows that urban school students can substantially improve their readiness for college by taking a more demanding sequence of core academic courses in high school. Urban students taking a more rigorous sequence of courses in mathematics and science and finding success in those courses score at or above national averages on the ACT. Regardless of gender, ethnicity, or family income, those students who elect to take four or more years of rigorous English courses and three or more years of rigorous course work in mathematics, science, and social studies earn higher ACT scores and are more successful in college than those who have not taken those courses (ACT & Council of Great City Schools, 1999). Subsequent research has substantiated these findings and confirmed the value of rigor in the core courses (ACT, 2004; ACT & The Education Trust, 2004).

Teach test-taking strategies. Students may be helped by being taught specific test-taking strategies, such as the following:

- Learn to pace yourself.
- Know the directions and understand the answer sheet.
- Read carefully and thoroughly.
- Answer easier questions first; skip harder questions and return to them later.
- Review answers and check work, if time allows.
- Mark the answer sheet quickly and neatly; avoid erasure marks on the answer sheet.
- Answer every question (you are not penalized for guessing on EXPLORE).
- Become familiar with test administration procedures.
- Read all the answer choices before you decide which is the correct answer.

Students are more likely to perform at their best on a test if they are comfortable with the test format, know appropriate test-taking strategies, and are aware of the test administration procedures. Test preparation activities that help students perform better in the short term will be helpful to those students who have little experience taking standardized tests or who are unfamiliar with the tests' formats.

Search out other sources of help. School personnel in urban or high-poverty middle schools can investigate programs such as GEAR UP, which "provides federal funds for schools to prepare low-income middle school students for high school and college preparation through multiple school reform efforts. School districts, colleges, community organizations, and businesses often form partnerships to provide teachers with enhanced professional development opportunities to ensure they have the necessary tools and strategies to teach middle school and high school effectively" (Noeth & Wimberly, 2002, p. 18).

What Do the EXPLORE Mathematics Test Results Indicate About Low-Scoring Students?

Students who score 13 or below on the EXPLORE Mathematics Test are likely to have some of the knowledge and skills described in the EXPLORE Mathematics College Readiness Standards for the 13–15 range. Low-scoring students may be able to demonstrate skills in a classroom setting that they are not able to demonstrate in a testing situation. Therefore, these students need to become more consistent in demonstrating these skills in a variety of contexts and situations.

The EPAS Mathematics College Readiness Standards indicate that students who score 13 or below tend to demonstrate some of the following skills:

- Perform one-operation computation with whole numbers and decimals
- Solve problems in one or two steps using whole numbers
- Perform common conversions (e.g., inches to feet or hours to minutes)
- Calculate the average of a list of positive whole numbers
- Perform a single computation using information from a table or chart
- Recognize equivalent fractions and fractions in lowest terms
- Exhibit knowledge of basic expressions (e.g., identify an expression for a total as $b + g$)
- Solve equations in the form $x + a = b$, where a and b are whole numbers or decimals
- Identify the location of a point with a positive coordinate on the number line
- Estimate or calculate the length of a line segment based on other lengths given on a geometric figure

Overall, these students will likely benefit from encouragement in performing calculations and solving equations involving real numbers; in extending their knowledge of graphing to the coordinate plane; in becoming more comfortable with the basic concepts of probability, statistics, and data analysis through real-world problems; and in extending measurement concepts to include perimeter and area for a variety of geometric figures.

What Does Research Say About How Mathematics Instruction Should Be Conducted?

Research suggests that learning is maximized when students take a demanding core curriculum and engage in rigorous learning activities. The core curriculum must be embedded in a learning environment where students are motivated to work hard. To be motivated, students need to see the relevance of their schoolwork (LaPoint, Jordan, McPartland, & Towns, 1996). Research also suggests that framing learning and performance tasks within contexts that are familiar cultural experiences may improve students' cognitive functioning and consequently their achievement (Boykin & Bailey, 2000).

National Council of Teachers of Mathematics (NCTM) recommendations for change in mathematics education call for teachers to use a wide range of instructional strategies:

> A variety of instructional methods should be used in classrooms to cultivate students' abilities to investigate, make sense of, and construct meanings from new situations; to make and provide arguments for conjectures; and to use a flexible set of strategies to solve problems from both within and outside mathematics. In addition to traditional teacher demonstrations and teacher-led discussions, greater opportunities should be provided for small-group work, individual explorations, peer instruction, and whole-class discussions in which the teacher serves as a moderator. (NCTM, 1989, pp. 125, 128)

What Can Be Done to Help Students Understand the Mathematical Situations They Encounter?

Students need to see how mathematics crosses into other disciplines and is used in real life, not just as isolated pieces of information to be used only in mathematics class. Today's mathematics instruction should not be just about memorization, though some things like the basic facts of arithmetic need to be internalized to the point of automatic recall. The memorization-and-drill approach does not provide an in-depth understanding of those mathematical concepts required to build a strong foundation in math, nor does the lecture method.

The "scaffolding" method, in which the teacher sets up interactive learning activities at increasingly challenging levels as students progress toward mastery of a concept, can be used as an alternative to drills and lecture. The learning activities can be organized so that the students are interacting with the teacher or with their peers. The teacher needs to monitor group discussions and guide the students as they use strategies of questioning, summarizing, clarifying, and predicting to achieve comprehension of the major concept(s) being taught.

For example, a teacher could increase the level of difficulty in algebraic equations by using a larger coefficient of the variable. Another idea would be to have students who have internalized how to solve algebraic equations work with others who still need assistance, by playing a game of "concentration." The teacher could make a set of cards with simple one-step equations and their solutions and have the students pair a one-step equation with the correct solution as a "match." The level of difficulty for the game could be increased by making another set of cards that includes a variety of one- and two-step equations. Each student takes a turn selecting cards until he or she gets a matched pair (one-step equations) or matched group (two-step equations). Play would continue until all of the cards were selected, and a player's standing within the group could be calculated by assigning points for each "match." Matched groups that involve two steps for their solution could be worth more points.

Mathematics study should include investigation of patterns in numbers, shapes, data, probability, and growth/decay, along with practice in communicating mathematically. Students should also be encouraged to guess:

> Part of developing student confidence is the realization that guessing plays an important role in learning mathematics. Even so, students must learn that the guesses must be tempered by validation and attempts to structure an explanation as to why the guess is an appropriate response to the situation at hand. Developing the ability to conjecture, test, revise, and reconjecture is an important step toward thinking mathematically. (Association for Supervision and Curriculum Development [ASCD] & The Education & Technology Resource Center, 1999, p. 9)

Today, mathematics requires students to move beyond memorization and drill and to develop skills that include problem solving, reasoning, representation, and communication. Students need to be able to visualize and make connections between the various branches of mathematics and between mathematics and other content areas. Students might need to solve a probability problem geometrically by using coordinates from a Cartesian plane, or to use their knowledge of number concepts, properties, and theorems (such as exponentiation, root extraction, and the Pythagorean theorem) along with their algebraic manipulation skills to solve a measurement problem involving perimeter, area, or volume of a composite geometric figure. Teachers need to guide their students through activities such as small-group work, individual explorations, use of technology, peer instruction, and math logs/journals so that the students have the opportunity to develop reasoning and problem-solving skills—skills they'll need to use when confronted with purely mathematical problems and in real-world situations with connections to other content areas.

Teachers also need to help students understand difficult or abstract concepts by using real-life examples to make the concepts more concrete. For example, the order of operations to solve two-step algebra equations could be reinforced using shoes and socks. When students get dressed they put on socks first, then shoes; but when they want to undress, they do the reverse: they remove instead of put on, removing their shoes first and socks second. In analyzing the equation $2x + 3 = 10$, the order of operations is to multiply x by 2 and then add 3. To solve the equation, the inverse operations are used in the reverse order: subtract 3 and then divide by 2.

Analyze the equation	→	get dressed
Multiply x by 2	→	put on socks
Add 3	→	put on shoes
Solve the equation	→	get undressed
Subtract 3	→	take off shoes
Divide by 2	→	take off socks

By connecting the abstract to the concrete as in the shoes and socks example, the teacher can reinforce mathematical concepts by relating them to situations in everyday life.

What Knowledge and Skills Are Low-Scoring Students Ready to Learn?

For students who score 13 or below on the EXPLORE Mathematics Test, their target achievement outcomes could be a combination of the College Readiness Standards listed in the 13–15 (on page 37) and 16–19 ranges (below). Additional information will need to be reviewed to determine which skills in the 13–15 range students can and cannot demonstrate. For example, if there are several skills students cannot demonstrate in the 13–15 range, students should focus on them first, and then possibly work on some from the 16–19 range as needed. The College Readiness Standards for the 16–19 range include the following:

- Solve routine one-step arithmetic problems (using whole numbers, fractions, and decimals) such as single-step percent
- Solve some routine two-step arithmetic problems
- Calculate the average of a list of numbers
- Calculate the average, given the number of data values and the sum of the data values
- Read tables and graphs
- Perform computations on data from tables and graphs
- Use the relationship between the probability of an event and the probability of its complement
- Recognize one-digit factors of a number
- Identify a digit's place value
- Substitute whole numbers for unknown quantities to evaluate expressions
- Solve one-step equations having integer or decimal answers
- Combine like terms (e.g., $2x + 5x$)
- Locate points on the number line and in the first quadrant
- Exhibit some knowledge of the angles associated with parallel lines
- Compute the perimeter of polygons when all side lengths are given
- Compute the area of rectangles when whole number dimensions are given

By no means should these be seen as limiting or exclusive goals. As stated earlier, it is important to use multiple sources of information to make instructional decisions and to recognize that individual students learn at different rates and in different sequences. What's important is to get students communicating mathematically.

What Strategies/Materials Can Teachers Use in Their Classrooms?

According to Bryan Goodwin, senior program associate at the Mid-continent Research Education Laboratory (McREL), "it is important to note that improving the performance of disenfranchised students does not mean ignoring other students. Indeed, many of the changes advocated—such as making curricula more rigorous and creating smaller school units—will benefit all students" (Goodwin, 2000, p. 6). Means and Knapp (1991) express a similar view:

> A fundamental assumption underlying much of the curriculum in America's schools is that certain skills are "basic" and must be mastered before students receive instruction on more "advanced" skills, such as reading comprehension, written composition, and mathematical reasoning. . . . Research from cognitive science questions this assumption and leads to a quite different view of children's learning and appropriate instruction. By discarding assumptions about skill hierarchies and attempting to understand children's competencies as constructed and evolving both inside and outside of school, researchers are developing models of intervention that start with what children know and provide access to explicit models of thinking in areas that traditionally have been termed "advanced" or "higher order." (p. 1)

Pages 44–55 exemplify the kind of teacher-developed activity that could be used in a classroom for all students, not just those who have scored low on a standardized assessment like EXPLORE. The activity has students solve some routine two-step arithmetic problems as they perform common conversions. The students are then asked to present and explain, using appropriate mathematical vocabulary, why and how they used unit fractions to complete the conversion. Two Student Activity Sheets and two Rating Scale Checklists are included that could be used by the teacher to assess student learning. Also included is a suggested Math Journal Checklist that could be used either by the teacher or student to evaluate entries in a Math Journal. The activity provides suggestions for related investigations that would enable the teacher to extend and build on the original ideas in order to reinforce and strengthen student learning.

How Is the Activity Organized?

A template for the instructional activity appears on page 43. Since the instructional activity has multiple components, an explanation of each is provided below.

A The primary *Mathematics Strands* are displayed across the top of the page.

B The *Guiding Principles* section consists of one or more statements about instruction, assessment, thinking skills, student learning, and other educationally relevant topics.

C The *Title* and *Subject Area(s)/Course(s)* information allows you to determine at a glance the primary focus of the activity and whether it might meet the needs of your student population.

D The *Purpose* statement describes knowledge and skills students may have difficulty with and what will be done in the activity to help them acquire that knowledge and skills.

E The *Overview* section provides a brief description of how the knowledge and skills listed in the purpose statement will be taught and suggests an estimated time frame for the entire activity.

F The *Links to College Readiness Standards* section indicates the primary knowledge and skills the activity will focus on. These statements are tied directly to the strands listed at the top of the page.

G The next section, *Description of the Instructional Activity,* is divided into three interrelated parts: Materials/Resources, Introduction, and Suggested Teaching Strategies/Procedures. The section provides suggestions for engaging students in the activity, and gives related topics and tasks. The activity addresses a range of objectives and modes of instruction, but it emphasizes providing students with experiences that focus on reasoning and making connections, use community resources and real-life learning techniques, and encourage students to ask questions—questions leading to analysis, reflection, and further study and to individual construction of meanings and interpretations.

H Valuable *Comments/Tips from Classroom Teachers* are provided for the activity. As the title indicates, this text box includes ideas from current classroom teachers.

I The *Suggestions for Assessment* section offers ideas for documenting and recording student learning. This section describes two types of assessments: Embedded Assessments and Summative Assessments. Embedded Assessments are assessments that inform you as to where your students currently are in the learning process (a formative assessment that is primarily teacher developed and is integral to the instructional process—at times the instruction and assessment are indistinguishable). The second type of assessment is a Summative Assessment (a final assessment of students' learning), which provides a description of the knowledge and skills students are to have mastered by the end of the activity and the criteria by which they will be assessed.

J The *Links to Ideas for Progress* section provides statements that suggest learning experiences (knowledge and skills to be developed) that are connected to the Suggested Strategies/Activities.

K The *Suggested Strategies/Activities* section provides a brief description of ways to reteach the skills or content previously taught or to extend students' learning.

This teacher-developed activity provides suggestions, not prescriptions. You are the best judge of what is necessary and relevant for your students. Therefore, we encourage you to review the activity, modifying and using those suggestions that apply, and disregarding those that are not appropriate for your students. As you select, modify, and revise the activity, you can be guided by the statements that appear in the Guiding Principles box at the beginning of the activity.

Linking Instruction and Assessment

Strand(s): A

Guiding Principles

- B
-

Title C

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Guiding Principles

- "Knowledge of students' understandings and ways of thinking helps teachers to construct worthwhile mathematical tasks." (National Council of Teachers of Mathematics [NCTM], 1991, p. 13)
- "Understanding develops as students construct new relationships among ideas, as they strengthen existing relationships among those ideas, and as they reorganize their ideas." (Secada, 1997, p. 8)
- "Assessment should support the learning of important mathematics and furnish useful information to both teachers and students." (NCTM, 2000, p. 22)

Converting Units of Measure

General Math or Pre-Algebra

Purpose

Working with conversions is difficult for many students. This activity can help students learn to convert from one standard unit to another by using unit fractions (that is, the numerator and denominator represent equal quantities).

Overview

After introducing or reviewing conversion units for length, capacity, and weight, have the students independently complete an activity sheet that focuses strictly on computation. Then have the students complete a second activity sheet (with a partner or in a small group) comprised of real-world scenarios that invite discussion among students as they decide how each unit fraction should be set up to perform each conversion. The approximate time for this activity is two to three class periods of 45 to 50 minutes each.

Links to College Readiness Standards

- Perform common conversions (e.g., inches to feet or hours to minutes)
- Solve some routine two-step arithmetic problems
- Solve multistep arithmetic problems that involve planning or converting units of measures (e.g., feet per second to miles per hour)

Description of the Instructional Activity

Materials/Resources

- Pencil
- Calculator (optional)
- Tape measures, rulers, and yardsticks
- Sand, kitty litter, rice, or water
- Containers of various capacity (c, pt, qt, gal)
- Exercise 1: Measure for Measure Student Activity Sheet (pp. 48–49)
- Exercise 2: Measuring Up Student Activity Sheet (pp. 50–52)
- *Optional Assessments*
  - ✓ Math Journal Checklist (p. 53)
  - ✓ Rating Scale Checklist 1 (p. 54)
  - ✓ Rating Scale Checklist 2 (p. 55)

Introduction—Though taught to multiply or divide to perform common conversions, many students have trouble choosing the appropriate operation for a given situation. Such students can gain a reliable technique for conversion by learning to use one or more unit fractions when multiplying, and realizing that the unit names (e.g., inches, gallons) need to cancel. Students provided with this method will become confident in their ability to convert to different units by using a method or algorithm that they understand.

Start the lesson by discussing why correct conversions are important (e.g., provide samples of lemonade made with the wrong amount of sugar because of an inaccurate conversion from a recipe). Discuss how errors could have serious consequences (e.g., a patient receives the wrong amount of medicine because of an inaccurate conversion by a pharmacist). Have students write an entry in their Math Journal that requires them to think of professions that use conversions on a daily basis. You could use a Math Journal Checklist to evaluate students' skills and understandings. Introduce or review conversion units for length, capacity, and weight (e.g., 12 in = 1 ft, 2 pt = 1 qt, 16 oz = 1 lb).

Suggested Teaching Strategies/Procedures— Measure several items in the classroom in inches using a tape measure, and then have students convert the measures into feet using a 12″ ruler or into yards using a yardstick to see how many rulers or yardsticks fit. Take a gallon of sand (or some other appropriate classroom material) and have the students convert the gallon to equivalent quarts, pints, or cups. Then use the examples on this page to help students determine and understand the appropriate operation and unit name for each conversion. When using the examples, point out that each unit fraction is equal to 1 (the numerator and denominator represent equal quantities, and a quantity divided by itself equals 1) and that multiplying a quantity by 1 changes only its form, not the amount it represents.

Comments/Tips from Classroom Teachers

Preselecting items for the students to measure and convert that are multiples of 12 for the feet conversions and multiples of 36 for the yards conversions will avoid fractional portions of a foot or yard.

Example #1: Convert 48 in = ___ ft

$$48 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}}$$

$$48 \cancel{\text{in}} \cdot \frac{1 \text{ ft}}{12 \cancel{\text{in}}}$$

$$\frac{48}{12} = 4 \text{ ft}$$

Example #2: Convert 6 gal = ___ qt

$$6 \text{ gal} \cdot \frac{4 \text{ qt}}{1 \text{ gal}}$$

$$6 \cancel{\text{gal}} \cdot \frac{4 \text{ qt}}{1 \cancel{\text{gal}}}$$

$$6 \cdot 4 = 24 \text{ qt}$$

Example #3: Convert 6 gal = ___ c

$$6 \text{ gal} \cdot \frac{4 \text{ qt}}{1 \text{ gal}} \cdot \frac{2 \text{ pt}}{1 \text{ qt}} \cdot \frac{2 \text{ c}}{1 \text{ pt}}$$

$$6 \cancel{\text{gal}} \cdot \frac{4 \cancel{\text{qt}}}{1 \cancel{\text{gal}}} \cdot \frac{2 \cancel{\text{pt}}}{1 \cancel{\text{qt}}} \cdot \frac{2 \text{ c}}{1 \cancel{\text{pt}}}$$

$$6 \cdot 4 \cdot 2 \cdot 2 = 96 \text{ c}$$

Exercise 1: Measure for Measure

Distribute the Measure for Measure Student Activity Sheet that can be found on pages 48–49. Have students independently perform the conversions and show their work (determine unit fraction(s), cancel out same unit names, and multiply/divide) below each problem. After the students have completed the activity, each student can present the solution for one of the problems from the activity on the overhead or whiteboard. Have the student explain to the class, using appropriate mathematical vocabulary, why and how each unit fraction was used to complete the conversion. Rating Scale Checklist 1 is provided as a suggestion to help you evaluate students' skills and understandings.

Exercise 2: Measuring Up

Distribute the Measuring Up Student Activity Sheet that can be found on pages 50–52. Assign the students to small groups (2–4 students). Some students may need to discuss within their group how to set up the unit fraction(s) for the conversion required in each real-world scenario. Have students perform the conversions and show their work (determine unit fraction(s), cancel out same unit names, multiply/divide, and label with correct unit) for each problem on a separate sheet of paper. After the students have completed the activity, each student can present the solution for one of the problems from the activity on the overhead or whiteboard. Have the student explain to the class, using appropriate mathematical vocabulary, why and how each unit fraction was used to complete the conversion. Rating Scale Checklist 2 is provided as a suggestion to help you evaluate students' skills and understandings.

Suggestions for Assessment

Embedded Assessment (Anecdotal Notes)—Observe students as they complete the activity worksheets, noting whether the students are actively participating in the mathematical calculations and accurately modeling their results as shown in Example #3.

Embedded Assessment (Math Journal Checklist)—Have students keep a journal that documents various types of math problems they encounter throughout the school year. Some entries could include sample problems with written explanations of the process(es) used to solve and draw an appropriate conclusion. To monitor students' ability to perform conversions, ask students to write a journal entry that explains how they would "teach" their classmates to make a given conversion. A Math Journal Checklist could be used to evaluate students' entries in their Math Journal. You and your students could generate the criteria for the checklist together. The checklist may be focused in various ways: it may cover a wide range of skills and understandings, it may cover a specific set of skills and understandings, or it may change as a unit unfolds to reflect new skills and understandings being learned. The checklist on page 53, for example, documents such broad skills as students' mathematical understandings, use of mathematical language, and mathematical reasoning or argumentation. It also documents students' abilities to make inferences and generalizations, to ask thoughtful questions, and to reflect on their thinking processes. The Math Journal could be used throughout the entire unit or math course.

You could use the checklist in several ways. For example, you could ensure students understand that the checklist is to be used during the entire unit and that all the criteria on the checklist should be illustrated *somewhere* in the journal, though certainly all would not be used in each entry. You might specify certain checklist criteria to be used in certain entries, or you could ask students to choose one or two criteria from the checklist to focus on as they write each journal entry. The checklist could be used both by you and your students to evaluate students' entries in their Math Journals.

Summative Assessment (Rating Scale Checklist)—Qualitative Rating Scale Checklists, such as the ones on pages 54–55, could be used to assess students' mathematical skills and understandings, how well they communicate their ideas, and the quality and accuracy of their work.

Enhancing Student Learning

Links to Ideas for Progress

- Use multiple operations to solve multistep arithmetic problems
- Solve routine arithmetic problems that involve rates, proportions, and percents
- Select and use appropriate units when solving problems that involve one or more units of measure

Suggested Strategies/Activities

If students have difficulty with either activity, let them work with a partner to calculate a smaller number of conversions that use easier numbers. Students might be given examples where the conversions could be performed using concrete materials. Having a ruler as a visual aid or having measuring cups to use can help some students with comparisons and understanding of conversions to equal measures.

Schedule a health care professional, a chef, or a carpenter for a classroom visit to discuss how and why they perform conversions on a daily basis. Students could use the Internet, read newspapers, or interview members of businesses in their local community to research/report how important conversions to equivalent measurements might be for different business and marketing situations. Students could discuss how conversions might be used on a family vacation (e.g., miles per gallon for a trip) or needed for a home-remodeling project (e.g., square yards of carpet needed to calculate the cost of materials). They could brainstorm, analyze, and draw conclusions about situations or occupations where performing accurate measurement conversions are crucial.

Exercise 1: Measure for Measure

Student Activity Sheet

Name: _____ Period: _____ Date: _____

Directions: Use one or more unit fractions to perform the following conversions.

Length	Capacity	Weight
12 inches (in) = 1 foot (ft)	8 fluid ounces (oz) = 1 cup (c)	16 dry ounces (oz) = 1 pound (lb)
3 ft = 1 yard (yd)	2 c = 1 pint (pt)	2,000 lb = 1 ton (T)
5,280 ft = 1 mile (mi)	2 pt = 1 quart (qt)	
	4 qt = 1 gallon (gal)	

Show your steps in the space below each problem. Be prepared to use appropriate mathematical vocabulary to explain to your classmates why and how you used each unit fraction to complete the conversion.

Example: Convert 6 gal = ___ qt

$$6 \text{ gal} \cdot \frac{4 \text{ qt}}{1 \text{ gal}}$$

$$6 \ \cancel{\text{gal}} \cdot \frac{4 \text{ qt}}{1 \ \cancel{\text{gal}}}$$

$$6 \cdot 4 = 24 \text{ qt}$$

---

1. 4 lb = _____ oz

2. 8,000 lb = _____ T

3. 12 qt = _____ gal

4. 36 in = _____ ft

5. 6 pt = _____ c

6. 10 gal = _____ pt

7. _____ pt = 3 qt

8. 4 mi = _____ ft

9. 63 ft = _____ yd

10. 12 yd = _____ ft

11. _____ oz = 3 c

12. 4 qt = _____ pt

13. 32 oz = _____ c

14. _____ yd = 27 ft

15. 16 T = _____ lb

16. _____ ft = 84 in

17. 7 c = _____ pt

18. $3\frac{1}{2}$ ft = _____ in

Bonus: 2 miles = _____ inches

Exercise 2: Measuring Up

Student Activity Sheet

Name: _____ Period: _____ Date: _____

Directions: Use one or more unit fractions to perform the following conversions.

Length	Capacity	Weight
12 inches (in) = 1 foot (ft)	8 fluid ounces (oz) = 1 cup (c)	16 dry ounces (oz) = 1 pound (lb)
3 ft = 1 yard (yd)	2 c = 1 pint (pt)	2,000 lb = 1 ton (T)
5,280 ft = 1 mile (mi)	2 pt = 1 quart (qt)	
	4 qt = 1 gallon (gal)	

Show your steps in the space below each problem. Be prepared to use appropriate mathematical vocabulary to explain to your classmates why and how you used each unit fraction to complete the conversion.

Example: Convert 6 gal = ___ c

$$6 \text{ gal} \cdot \frac{4 \text{ qt}}{1 \text{ gal}} \cdot \frac{2 \text{ pt}}{1 \text{ qt}} \cdot \frac{2 \text{ c}}{1 \text{ pt}}$$

$$6 \cancel{\text{gal}} \cdot \frac{4 \cancel{\text{qt}}}{1 \cancel{\text{gal}}} \cdot \frac{2 \cancel{\text{pt}}}{1 \cancel{\text{qt}}} \cdot \frac{2 \text{ c}}{1 \cancel{\text{pt}}}$$

$$6 \cdot 4 \cdot 2 \cdot 2 = 96 \text{ c}$$

1. Jerry's height is 72 inches; how tall is he in feet?

2. Carlos can spit a watermelon seed 12 feet; how many yards is this?

3. The pilot announces that you are flying at an altitude of 36,960 feet; how many miles is this?

4. A slug crawls 4 yards toward your garden; how many inches did it move?

5. Your fence is 24 yards long; how many feet is this?

6. You need 20 quarts of ice cream for a class party; how many gallons should you buy?

7. How many gallons are there in 64 ounces of orange juice?

8. Your truck weighs 5,000 pounds; how many tons is this?

9. Your room has a wall that is 3 yards long; how many feet is this? How many inches?

10. You have 8 gallons of apple cider. You are going to store it in quart containers. How many containers will you need to complete the task?

11. The table in the front of the class is 5 feet long; how many inches is this?

12. How many ounces are in 2 gallons of juice? How many 8-ounce glasses will it fill?

13. A coffee thermos holds 1 quart. How many thermoses will be needed to take 2 gallons of coffee to the football game?

14. There are 96 ounces of chocolate syrup to sell at the ice cream social. If it is poured into 1-cup containers and then sold, how many containers will you need so that you can sell all of the chocolate syrup?

15. Laticia told her friends she weighs 1,680 ounces. What is her weight in pounds?

16. An elevator sign says it will hold 3,000 pounds; how many tons will it hold?

17. The hallway is 6 feet wide; how many yards is this?

18. Mr. Brown's recipe calls for 2 cups of milk. It serves 8 people. How many gallons of milk will he need to serve 64 people?

19. Sally needs to fill twenty-four 8-ounce containers to the top with salsa. How many gallons of salsa will she need?

20. If Sally bought a gallon and a quart of salsa, would she have enough to complete her task in question 19? Explain/justify your answer.

Bonus: Write a problem depicting a real-life scenario that requires a conversion to obtain the solution.

Math Journal Checklist

Name: _____ Period: _____ Date: _____

(The blanks at the bottom are for additional criteria generated by the teacher and/or students.)

Criteria	**Yes/No** (Include dates that journal is reviewed.)	**Comments**
Makes connections between mathematics and other disciplines or applied fields		
Generally uses precise mathematical language and notation		
Explains own thinking; may use mathematical models, drawings, facts, etc.		
Provides both complete and accurate mathematical information		
Uses logical reasoning to make conjectures, to construct arguments, and/or to validate and prove conclusions		
Comments on arguments, recognizing fallacies or underlying assumptions		
Makes inferences and/or generalizations based on logic and probability		
Analyzes problems, selecting various solution strategies and tools		
Poses thoughtful questions		
Reflects on and assesses own thinking and learning		
Writes about mathematical materials read with understanding		
Analyzes new information gained		

Summative Assessment—Rating Scale Checklist 1

Name: _____ Period: _____ Date: _____

Directions: Note the degree of evidence the student has demonstrated for each criterion, tallying subscores for each.

Exercise 1: Measure for Measure Criteria and Scoring				
Accuracy of Work	**Exemplary Evidence 4**	**Much Evidence 3**	**Some Evidence 2**	**Little Evidence 1**
■ Selects appropriate unit fraction so unit names cancel				
■ Calculations are completed accurately				
■ Conversion is completed accurately				
			Subscore:	
Mathematical Skills and Understandings	**Exemplary Evidence 4**	**Much Evidence 3**	**Some Evidence 2**	**Little Evidence 1**
■ Demonstrates understanding of how to cancel unit names				
■ Demonstrates understanding of how to set up multistep conversions to produce the desired unit				
			Subscore:	
Organization of Information	**Exemplary Evidence 4**	**Much Evidence 3**	**Some Evidence 2**	**Little Evidence 1**
■ Has completed the Student Activity Sheet				
■ Has shown computation work/steps				
			Subscore:	
Communication of Ideas	**Exemplary Evidence 4**	**Much Evidence 3**	**Some Evidence 2**	**Little Evidence 1**
■ Provides a well-reasoned explanation of why unit fraction conversions make equivalent units				
■ Provides a well-reasoned explanation of how each unit fraction makes the conversion complete				
■ Uses appropriate mathematical vocabulary to express thinking				
			Subscore:	

Total Score for Exercise 1 _____

Summative Assessment—Rating Scale Checklist 2

Name: _____ Period: _____ Date: _____

Directions: Note the degree of evidence the student has demonstrated for each criterion, tallying subscores for each.

Exercise 2: Measuring Up Criteria and Scoring				
Accuracy of Work	**Exemplary Evidence 4**	**Much Evidence 3**	**Some Evidence 2**	**Little Evidence 1**
■ Selects appropriate unit fraction so unit names cancel				
■ Calculations are completed accurately				
■ Conversion is completed accurately				
■ Answers use the proper unit names				
			Subscore:	
Mathematical Skills and Understandings	**Exemplary Evidence 4**	**Much Evidence 3**	**Some Evidence 2**	**Little Evidence 1**
■ Demonstrates understanding of how to cancel unit names				
■ Demonstrates understanding of how to set up multistep conversions to produce the desired unit				
			Subscore	
Organization of Information	**Exemplary Evidence 4**	**Much Evidence 3**	**Some Evidence 2**	**Little Evidence 1**
■ Has completed the Student Activity Sheet				
■ Has included explanation/justification for #20				
■ Has shown computation work/steps				
			Subscore	
Communication of Ideas	**Exemplary Evidence 4**	**Much Evidence 3**	**Some Evidence 2**	**Little Evidence 1**
■ Provides a well-reasoned explanation of why unit fraction conversions make equivalent units				
■ Provides a well-reasoned explanation of how each unit fraction makes the conversion complete				
■ Uses appropriate mathematical vocabulary to express thinking				
			Subscore	

Total Score for Exercise 2 _____

Instructional Activities for EXPLORE Mathematics

Why Are Additional Instructional Activities Included?

The set of instructional activities that begins on page 58 was developed to illustrate the link between classroom-based activities and the skills and understandings embedded in the EXPLORE Mathematics Test questions. The activities are provided as examples of how classroom instruction and assessment, linked with an emphasis on reasoning, can help students practice skills and understandings they will need in the classroom and in their lives beyond the classroom. It is these skills and understandings that are represented on the EXPLORE Mathematics Test.

A variety of thought-provoking activities, such as applying mathematics, making connections within mathematics and to other areas, small- and large-group discussions, and both independent and collaborative activities, are included to help students develop and refine their skills in many types of situations.

The instructional activities that follow have a similar organizational structure as the one in the previous section. *Like the other activity, these activities were not developed to be a ready-to-use set of instructional strategies.* ACT's purpose is to illustrate how the skills and understandings embedded in the EXPLORE Mathematics Test questions can be incorporated into classroom activities.

For the purpose of this part of the guide, we have tried to paint a picture of the ways in which the activities could work in the classroom. We left room for you to envision how the activities might best work for you and your students. We recognize that as you determine how best to serve your students, you take into consideration your teaching style as well as the academic needs of your students; state, district, and school standards; and available curricular materials.

The instructional activities are not intended to drill students in skills measured by the EXPLORE Mathematics Test. It is never desirable for test scores or test content to become the sole focus of classroom instruction. However, considered with information from a variety of other sources, the results of standardized tests can help you identify areas of strength and weakness. The activities that follow are examples of sound educational practices and imaginative, integrated learning experiences. As part of a carefully designed instructional program, these activities may result in improved performance on the EXPLORE Mathematics Test—not because they show how to drill students in specific, isolated skills but because they encourage thinking and integrated learning. These activities can help because they encourage the kind of thinking processes and strategies the EXPLORE Mathematics Test requires.

Guiding Principles

- "Powerful learning . . . comes from developing true understanding of concepts and higher order thinking associated with various fields of inquiry and self-monitoring of thinking." (Zemelman, Daniels, & Hyde, 1993, p. 8)
- "Although assessment is done for a variety of reasons, its main goal is to advance students' learning and inform teachers as they make instructional decisions." (NCTM, 1995, p. 13)

Algebraic Representations

College Readiness Standards

- Exhibit knowledge of basic expressions (e.g., identify an expression for a total as $b + g$)
- Add and subtract simple algebraic expressions
- Perform straightforward word-to-symbol translations
- Solve real-world problems using first-degree equations

Description of the Instructional Activity

The teacher could begin class with a number-guessing game in which students think of a number and follow the teacher's instructions to perform a series of operations (possibly without the aid of pencil and paper or a calculator); and then the teacher correctly guesses their number. For example, the teacher could give the following instructions: pick a number between 1 and 10, add 5 to it, double the result, subtract 4, divide by 2, and subtract your starting number; then the teacher correctly guesses the result as 3. After several games, the teacher could lead the class in a discussion of why everyone ended with the same number. The operations could be listed on the board and the process modeled, using a variable to represent the starting number.

The game could be modified so the result is a given multiple of the starting number or a predetermined difference from the starting number. Students could be given an intended result and be asked to develop a set of instructions that would yield that result.

The class could discuss words that typically describe mathematical operations (e.g., *increased by, sum, less than, triple, or half*). Students could practice translating between English phrases and algebraic expressions. These phrases could also be modeled using manipulatives such as algebra tiles.

Suggestions for Assessment

Journal—Because this lesson evolves over time, students could be asked to keep a journal to document the various types of expressions, equations, and problems they encounter. Among the entries students could record in their journal are descriptions of processes they tried, sample problems with written explanations of the process(es) used to solve them, and pictorial and symbolic representations of problems and their solutions. On occasion, the teacher could ask students to model and/or solve a particular problem to be included in their journal. The teacher could then verify the accuracy of the model and/or solution, correcting any misunderstandings.

Anecdotal Notes—Students could be observed as they model equations. The teacher could note whether students are actively participating and accurately modeling/solving the equations. The teacher could also note the cognitive level at which students are working (i.e., using manipulatives, pictorial representations, or symbols).

Ideas for Progress

- Do multistep computations with rational numbers
- Create expressions that model mathematical situations using combinations of symbols and numbers
- Identify, interpret, and generate symbolic representations that model the context of a problem

Suggested Strategies/Activities

The number-guessing game could also be modified to model equations (e.g., "When a number is doubled and then increased by 10, the result is 24"). Working in pairs, students could model and solve equations in the form $ax + c = dx + e$ by using algebra tiles, drawing diagrams, balancing items on a pan balance, or using symbols. At first, one or more of the constants could be equal to 0. Students could then progress to equations that require more steps to solve.

After students have developed an intuitive process, they could compare and discuss in small groups the process they used to solve equations. The teacher could model the process using symbols. Students could then practice solving equations that model problems in either purely mathematical or real-world settings.

Students could also be given expressions or equations and asked to write a real-world problem that the expression or equation could be modeling.

Guiding Principles

- "The mathematics curriculum should include exploration of statistics in real-world situations so that students can make inferences and convincing arguments that are based on data analysis." (NCTM, 1989, p. 105)
- "Together, assessment and instruction can build on students' understanding, interests, and experiences." (NCTM, 1995, p. 48)

Representing Data

College Readiness Standards

- Read tables and graphs
- Perform computations on data from tables and graphs
- Translate from one representation of data to another (e.g., a bar graph to a circle graph)
- Manipulate data from tables and graphs

Description of the Instructional Activity

Students could be shown a wide range of graphs and data representations that are accurately depicted in newspapers, magazines, and almanacs. Among the representations that could be focused on are bar graphs, line graphs, circle graphs, histograms, stem-and-leaf plots, and box-and-whisker plots.

Some sets of data could be displayed using several representations. Students could then be asked to identify which displays represent the data in an easily understood manner and to defend their response. Students could be given a series of examples in which the data could have been displayed in a better way and be asked to represent the data in another way. They could also discuss why some data does not lend itself to certain types of displays (e.g., using circle graphs to display Chicago's population as reported in the last five censuses).

Students could be given pairs of displays of the same data, in which one of the data representations is misleading (Huff, 1982). They could discuss the different features of the representations, such as scale, and how those can help, or hinder, the interpretation of data.

Suggestions for Assessment

Performance Task—Working in groups, students could be given a set of data or be asked to locate a set of data matching a certain criterion, using reference books or the Internet. Students could then display the data in the most appropriate way. They could also explain, verbally or in writing, why their display was the best choice and why other displays for their data were not chosen.

Checklist—A list of target outcomes could be devised. During class discussions and guided practice times, the teacher could monitor students' learning, marking progress on those outcomes that each student demonstrates. Comments regarding misunderstandings and partial knowledge could also be noted. The monitoring could take place over several days with the teacher modifying the lessons based on those targeted skills and understandings students are not demonstrating.

Ideas for Progress

- Interpret data from a variety of displays and use it in computation (e.g., mean, median, mode, range)
- Interpret data from a variety of displays (e.g., box-and-whisker plot) and use it along with additional information to solve real-world problems
- Gather, organize, display, and analyze data in a variety of ways to use in problem solving

Suggested Strategies/Activities

Students could use the data displayed in several different graphs to compute measures of central tendency. Students could then discuss which measure of central tendency would be the most appropriate statistical interpretation for each graph and why. They should understand that each measure of central tendency has merits in different situations.

Students could research other types of visual displays of data, using books by Tufte and Huff.

Guiding Principles

- "[Students should] understand and apply reasoning processes, with special attention to spatial reasoning and reasoning with proportions and graphs." (NCTM, 1989, p. 81)
- "In order to develop mathematical power in all students, assessment needs to support the continued mathematics learning of each student." (NCTM, 1995, p. 6)

Comparing Values

College Readiness Standards

- Solve routine two-step or three-step arithmetic problems involving concepts such as rate and proportion, tax added, percentage off, and computing with a given average
- Estimate or calculate the length of a line segment based on other lengths given on a geometric figure
- Locate points in the coordinate plane

Description of the Instructional Activity

The teacher could have the class brainstorm about different types of comparisons of two numbers or values (e.g., distance per a length of time, ratio of boys to girls in the class) and discuss when and why the numbers are compared. The class could also explore how the rate or ratio of one set of numbers could be used to find an equivalent rate or ratio.

Pairs of students could use manipulatives to explore several relationships to find the constant of proportionality (e.g., comparing the diameter of a circular object to its circumference or comparing lengths of corresponding sides of similar figures, using a figure and its image on an overhead screen or on scaled photocopies). Students could use these constants or proportions to determine an unknown value given one of the values (e.g., estimate the size of something unknown in the picture such as Godzilla's toe based on the size of something familiar in the picture).

The class could discuss different methods used to solve proportions and then conjecture when one method would be preferable to the other(s) (e.g., using cross-products, computing the constant of proportionality). Students could practice using the various methods.

Linking Instruction and Assessment

Strands: *Basic Operations & Applications; Graphical Representations; Measurement*

Suggestions for Assessment

Multiple-Choice Questions—Students could be given a set of problems to solve, such as computing values given straightforward proportions, identifying equivalent rates, and computing a constant of proportionality given a set of data. Students could provide written justifications for their answers.

Ideas for Progress

- Locate and describe objects in terms of their position on the number line and on a grid
- Sketch and identify line segments, midpoints, intersections, and vertical and horizontal lines
- Apply and use number properties to model and solve problems that involve reasoning with proportions

Suggested Strategies/Activities

Students could be given real-world problems that require reasoning with proportions (e.g., computing the better buy; using known rates such as gas mileage or speed limits to plan a trip; using similar figures to measure objects indirectly).

The idea of rate could be applied to graphs in the coordinate system. For example, the rate of change (slope) of lines could be explored. Students could devise their own algorithm for computing the slope of the line. The class could compare the intuitive algorithms and develop the slope formula with the teacher's guidance.

Linking Instruction and Assessment

Strands: *Numbers: Concepts & Properties; Expressions, Equations, & Inequalities; Graphical Representations*

Guiding Principles

- "The mathematics curriculum should include explorations of algebraic concepts and processes so that students can represent situations and number patterns with tables, graphs, verbal rules, and equations and explore the interrelationships of these representations." (NCTM, 1989, p. 102)
- "Perceptive questioning helps all students explain what they know and can do." (NCTM, 1995, p. 47)

Foundations of Functions

College Readiness Standards

- Exhibit knowledge of elementary number concepts including rounding, the ordering of decimals, pattern identification, absolute values, primes, and greatest common factor
- Substitute whole numbers for unknown quantities to evaluate expressions
- Locate points in the coordinate plane

Description of the Instructional Activity

Students could be given lists of numbers that form a pattern (e.g., 100, 97, 94, 91, . . .) or a situation that forms a sequence (e.g., the cost of purchasing an increasing number of an item). Students could be asked to generate the next several terms and the rule, using words and/or algebraic notation. Students then could be given rules for other sequences, both algebraic and with words, and asked to generate several terms for each.

Tables of values could be provided where the terms are not consecutive. Students could compute the functional values for known values or state the rule using words and algebraic expressions. For example, students could state the rule for the following table algebraically ($2x + 1$) or in words (multiply x by 2 and add 1).

Value of x	0	2	4	6	9
Rule ?	1	5	9	13	19

Ordered pairs representing the known values and their corresponding functional values could be graphed in the coordinate plane. Students could practice translating among the various representations.

Students could be given the graphs and rules for several sets of linear functions that have a recognizable pattern (e.g., add an amount to x, a multiple of x). Students could use the various patterns they identify to conjecture about linear functions, $f(x) = ax + b$. As the groups discuss their conjectures, the teacher could introduce the mathematical terms, notation, and ideas related to their generalizations.

Suggestions for Assessment

Rating Scale—Students could describe the conclusions they have made regarding the various representations of functions and how the representations are related. The teacher could rate students' entries using a scale of 1 to 4 on such criteria as quality of work, accuracy of work, mathematical skills and understanding, organization of information, and communication of ideas.

Pencil-and-Paper Tasks—Students could be given sequences or representations of functions in a variety of forms (e.g., list of terms, algebraic rule, or written description) and asked to write the sequences or functions in other ways.

Linking Instruction and Assessment

Strands: *Numbers: Concepts & Properties; Expressions, Equations, & Inequalities; Graphical Representations*

Ideas for Progress

- Apply elementary number concepts including identifying patterns pictorially and numerically (e.g., triangular numbers, arithmetic and geometric sequences), ordering numbers, and factoring
- Use the inverse relationships for the basic operations of addition and subtraction to determine unknown quantities
- Graph linear equations and inequalities, determine slopes of lines, identify parallel and perpendicular lines, and find distances

Suggested Strategies/Activities

Students could be given pictorial representations of sequences such as triangular numbers and then asked to generate the next few terms and describe the rule for the sequence in words. Another sequence that is easy to describe in words, but not algebraically, is the Fibonacci sequence. Students could generate terms, describe the rule, and research and report about the Fibonacci sequence and its connection to objects in nature.

If graphing calculators are available, students could enter sets of ordered pairs representing linear functions into the calculator and translate among the symbolic, tabular, and graphical representations. Groups of students could also use the table and graphing features of the calculator to explore nonlinear functions (e.g., functions based on the areas of squares and circles).

Putting the Pieces Together

ACT developed this guide to show the link between the EXPLORE Mathematics Test results and daily classroom work. The guide serves as a resource for teachers, curriculum coordinators, and counselors by explaining what the College Readiness Standards say about students' academic progress.

The guide explains how the test questions on the EXPLORE Mathematics Test are related to the College Readiness Standards and describes what kinds of reasoning skills are measured. The sample instructional activities and classroom assessments suggest some approaches to take to help students develop and apply their reasoning skills.

Where Do We Go From Here?

ACT recognizes that teachers are the essential link between instruction and assessment. We are committed to providing you with assistance as you continue your efforts to provide quality instruction.

ACT is always looking for ways to improve its services. We welcome your comments and questions. Please send them to:

College Readiness Standards Information Services
Elementary and Secondary School Programs (32)
ACT
P.O. Box 168
Iowa City, IA 52243-0168

"A mind, stretched to a new idea, never goes back to its original dimensions."

— Oliver Wendell Holmes

What Other ACT Products and Services Are Available?

In addition to the College Readiness Standards Information Services, ACT offers many products and services that support school counselors, students and their parents, and others. Here are some of these additional resources:

ACT's Website—**www.act.org** contains a host of information and resources for parents, teachers, and others. Students can visit **www.explorestudent.org**, which is designed to aid students as they prepare for their next level of learning.

The ACT—a guidance, placement, and admissions program that helps students prepare for the transition to postsecondary education while providing a measure of high school outcomes for college-bound students.

PLAN—a comprehensive assessment program designed to improve tenth-grade students' postsecondary planning and preparation and to enable schools to assist students and their parents in this important process.

WorkKeys®—a system linking workplace skill areas to instructional support and specific requirements of occupations.

ACT Online Prep™—an online test preparation program that provides students with real ACT tests and an interactive learning experience.

The Real ACT Prep Guide—the official print guide to the ACT, containing three practice ACTs.

DISCOVER®—a computer-based career planning system that helps users assess their interests, abilities, experiences, and values, and provides instant results for use in investigating educational and occupational options.

BIBLIOGRAPHY

This bibliography is divided into three sections. The first section lists the sources used in describing the EXPLORE Program, the College Readiness Standards for the EXPLORE Mathematics Test, and ACT's philosophy regarding educational testing. The second section, which lists the sources used to develop the instructional activities and assessments, provides suggestions for further reading in the areas of thinking and reasoning, learning theory, and best practice. The third section provides a list of resources suggested by classroom teachers.

(Please note that in 1996 the corporate name "The American College Testing Program" was changed to "ACT.")

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