Chapter 3
Mathematics

State Standards

All state standards documents include a common core of standards within mathematics influenced by the National Council of Teachers of Mathematics' Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) and Principles and Standards for School Mathematics (NCTM, 2000). While the state standards differ in emphasis and manner of organization, they typically include mathematical reasoning, problem solving, and integration of mathematics topics. The skills and knowledge measured on ACT's EPAS Mathematics Tests align well with the commonalties of skills and knowledge covered within the state standards documents and also with the National Assessment of Educational Progress Mathematics Framework. In other words, the design of the Mathematics Tests, which was based on ACT's Project Silver research project of the 1980s and has been regularly reviewed and refined in light of subsequent ACT National Curriculum Surveys, continues to accurately reflect school curricula.

Educator Surveys

ACT developed four Mathematics surveys and sent them to a total of 5,296 mathematics teachers (middle school/junior high, high school, and college) and department chairs (middle school/junior high and high school). The primary source for these samples was Market Data Retrieval (MDR), a company specializing in the education market. Survey recipients were sampled from the MDR database using selection criteria provided by ACT to ensure that a variety of geographic regions and schools were represented. Further criteria were based on whether the recipient taught one of the eligible courses, or, for the fourth sample, whether the recipient was a mathematics chair in middle school/junior high or high school. Table 3.1 lists the Mathematics courses used as selection criteria. ACT mailed curriculum surveys to the number of teachers and department chairs indicated in the table.

Table 3.1
Mathematics Courses Used as MDR Sample Selection Criteria

Sample	Courses	Sample Size
Middle School/Junior High	*Mathematics, Pre-Algebra, Algebra, Geometry*	1,197
High School	*Mathematics, Pre-Algebra, Algebra, Pre-Calculus, Calculus, Geometry, Statistics, Trigonometry*	2,400
Postsecondary	*Finite Math, Introduction to College Math, Discrete Math, Introduction to Probability, Introduction to Statistics, Probability and Statistics*	400
	Algebra, College Algebra, Precalculus, College Geometry, Geometry	400
	Calculus and Analytic Geometry, Calculus and Functional Analysis, Calculus I, II, and III	400
Department Chairs	*Middle School/Junior High Mathematics, High School Mathematics*	499

Table 3.2
Mathematics Survey Types and Response Rates

Survey type	Number mailed	Number returned	Response rate
Middle School/Junior High	1,197	232	19%
High School	2,400	472	20%
Postsecondary	1,200	324	27%
Department Chairs	499	146	29%
Total	5,296	1,174	22%

A total of 1,174 surveys were returned completed, yielding an overall response rate of 22% for the four surveys. The response rates varied slightly across the four sample groups. Response rates for all four surveys are listed in Table 3.2.

The survey respondents included individuals from all 50 states and the District of Columbia. The representation of the total respondent pool by region was 30.7% from the East, 32.5% from the Midwest, 16.2% from the Southwest, and 20.6% from the West. Analyses of the middle school/junior high, high school, and postsecondary respondent pools determined that they represented a wide variety of geographic locations and institutions. The department chair respondent pool, however, was found to be biased toward smaller schools. For this reason, survey results for the department chairs are not included in the discussion that follows.

Respondents to the Mathematics surveys were asked to consider lists of process and content skills. Middle school/junior high and high school respondents were asked to indicate at what grade level each skill was first taught (introduced) to the typical college-bound student in their school districts. All respondents were asked to indicate the level of importance of each skill on a scale of 1 to 5, where 1 represented *not important,* 3 represented *moderately important,* and 5 represented *very important.* Middle school/junior high teachers were asked to rate the importance of each skill as preparation for high school college-prep courses. High school teachers were to rate each skill as preparation for college. Finally, postsecondary faculty were to rate the importance of each skill as a prerequisite for entry-level coursework for one particular course—the course they taught that had the highest percentage of freshmen.

Middle school/junior high respondents were asked to consider a list of 65 content skills that span the core content covered in middle school/junior high math courses. High school and postsecondary respondents were asked to consider a list of 83 content skills that span the range of content covered in high school and college entry-level math courses (a broader range than is covered by the ACT Assessment Mathematics Test). Both lists of skills were identical to the lists of content skills used in the 1998 survey.

All respondents were also asked to consider a list of 20 process skills related to the processing of mathematical information (e.g., *Choosing an appropriate method for calculating, Using estimation to approximate solutions*) that also were listed in the 1998 survey.

The Mathematics surveys also asked respondents a variety of background questions. The middle school/junior high respondents were asked to indicate the grade level of the students they taught and which of the course types they taught (7th-grade *General Math,* 8th-grade *General Math, Pre-Algebra, Algebra,* and/or *Geometry*). High school respondents were also asked to indicate the grade level of their students and which of the course types they taught (*Algebra, Geometry, Advanced Algebra, Trigonometry, Pre-Calculus/Calculus, Probability/Statistics,* and/or *Discrete Math*). Both middle school/junior high and high school respondents were also asked how many years they had been teaching, the title of the primary textbook series they used, and a series of questions about the use of calculators in their classes and the types of calculators students used. Postsecondary mathematics faculty responded about the course they taught that had the highest percentage of freshmen, listing the course type (*Algebra, Geometry, Pre-Calculus, Calculus, Probability/Statistics,* or *Finite/Discrete Math*) and the course level (remedial, entry-level, or accelerated/honors/advanced placement). They were also asked to name the primary textbook they used, whether they permitted or required the use of calculators in the course, and what type of calculator they permitted/required.

The **middle school/junior high teachers** indicated that all 20 of the process skills were at least moderately important as preparation for college-prep coursework in high school, with 15 of the process skills rated 4.0 or higher. Those process skills receiving the lowest ratings tended to be ones involving theorems and proofs and more sophisticated use of

calculators. There were 11 of the process skills introduced before Grade 8 in at least 70% of the school systems represented, and 14 of the process skills introduced before Grade 9 in at least 70% of the school systems. The rank order of the process skills was essentially the same as in 1998.

The middle school/junior high teachers rated each of the 65 content skills as at least moderately important, with 47 of the content skills rated 4.0 or higher. There were 21 of the content skills introduced before Grade 8 in at least 70% of the schools systems represented, and 41 of the content skills introduced before Grade 9 in at least 70% of the school systems. The ranking of the content skills is very close to that in 1998. Only 5 skills changed by at least 10 places in the rankings. The 4 skills becoming more important are *Finding the slope of a line, Finding the median and mode, Solving systems of two linear equations in two variables algebraically,* and *Working with equations of parallel and perpendicular lines.* The content skill *Classifying plane geometric figures* became less important in the rankings by 10 places.

Approximately 40% of respondents require calculators in one of their classes, 55% make calculators optional in one of their classes, and 25% do not permit calculators in one of their classes. These percentages do not add up to 100% because teachers sometimes have different calculator policies for the different classes that they teach. The most common type of calculator recommended/required is a scientific calculator, with significant percentages also for 4-function calculators and graphing calculators. Table 3.8 summarizes calculator-related responses for the middle school/junior high teachers.

The **high school teachers** indicated that all 20 of the process skills were at least moderately important as preparation for college mathematics courses, with 14 of the process skills rated 4.0 or higher. Those process skills receiving the lowest ratings tended to be the ones involving theorems and proofs, manipulatives, and more sophisticated uses of calculators. There were 12 of the process skills introduced before Grade 10 in at least 70% of the school systems represented and 18 of the process skills introduced before Grade 12 in at least 70% of the school systems. The rank order of the process skills was essentially the same as in 1998. A notable exception was *Recalling quickly basic facts, definitions, formulas, and algebraic procedures then using them correctly to solve a problem,* which went from 13th in 1998 to 5th in 2003.

The high school teachers rated each of the 83 content skills as at least moderately important, with 41 of the content skills rated 4.0 or higher. There were 15 of the content skills introduced before Grade 10 in at least 70% of the school systems represented and 66 of the content skills introduced before Grade 12. The ranking of the content skills is very close to that in 1998. Only 8 skills changed by at least 10 places in the rankings. The 6 skills becoming more important are *Solving quadratic equations by factoring; Performing polynomial long division; Using the discriminant; Finding the mean, median, and mode; Determining the line of best-fit by eye for a set of data;* and *Working with correlations.* The 2 skills that became less important by at least 10 places are *Using the midpoint formula* and *Working with sigma notation.*

Approximately 65% of respondents require calculators in one of their classes, 40% make calculators optional in one of their classes, and 5% do not permit calculators in one of their classes. These percentages do not add up to 100% because teachers sometimes have different calculator policies for the different classes that they teach. The most common type of calculator recommended/required is a graphing calculator, with a significant percentage for scientific calculators and less than 10% for basic 4-function calculators. Table 3.8 summarizes calculator-related responses for the high school teachers.

The **college faculty,** each having chosen a specific entry-level course on which to base their responses, indicated that 11 of the 20 process skills were at least moderately important as prerequisite for the course they chose. Those process skills receiving the lowest ratings tended to be the ones involving constructing proofs, using manipulatives and/or pictorial representations, and using the more sophisticated capabilities of calculators. The ranking of the process skills was essentially the same as in 1998. Notable exceptions were *Recalling quickly basic facts, definitions, formulas, and algebraic procedures then using them correctly to solve a problem,* which went from 8th in 1998 to 2nd in 2003, and *Applying theorems to solve a problem,* which went from 17th to 12th.

The college faculty rated 25 of the 83 content skills as at least moderately important, with 8 of the content skills rated 4.0 or higher. The ranking of the content skills is very close to that in 1998. Only 4 skills changed by at least 10 places in the rankings. The 2 becoming more important are *Understanding continuity* and *Determining maxima, minima, and points of inflection for functions.* The 2 skills that became less important by at least 10 places are *Using counting techniques* and *Working with Venn diagrams.*

Approximately 50% of respondents require calculators in the course on which they based their responses, 40% make calculators optional in that course, and 10% do not permit calculators in that course. The most common type of calculator recommended/required is a graphing calculator, with a significant percentage for scientific calculators and less than 10% for basic 4-function calculators. Table 3.8 summarizes calculator-related responses for the college faculty.

Tables 3.3 and 3.4 present the 5 process skills and the 12 content skills, respectively, receiving the highest mean importance ratings by respondent group. A skill is italicized if all 3 groups ranked it high enough to be in the table. See Tables 3.9 and 3.10 for the full response data.

Table 3.3

Mathematics Process Skills Receiving the Highest Mean Importance Ratings, by Respondent Group

Rank (1 = most important)	Middle school/ junior high school teachers	High school teachers	College faculty
1	*Solving problems posed in real-world settings and interpreting the solution*	*Planning and carrying out a strategy for solving multistep problems*	Performing basic operations with a calculator
2	Reading and interpreting graphs, charts, and other data representations	*Solving problems posed in real-world settings and interpreting the solution*	Recalling quickly basic facts, definitions, formulas, and algebraic procedures then using them correctly to solve a problem
3	*Planning and carrying out a strategy for solving multistep problems*	Performing basic operations with a calculator	*Planning and carrying out a strategy for solving multistep problems*
4	Recognizing and using patterns to solve problems	Reading and interpreting graphs, charts, and other data representations	*Solving problems posed in real-world settings and interpreting the solution*
5	Applying mathematical ideas to new contexts	Recalling quickly basic facts, definitions, formulas, and algebraic procedures then using them correctly to solve a problem	Recognizing and using patterns to solve problems

Table 3.4

Mathematics Content Skills Receiving the Highest Mean Importance Ratings, by Respondent Group

Rank (1 = most important)	Middle school/ junior high school teachers	High school teachers	College faculty
1	*Performing addition, subtraction, multiplication, and division on signed rational numbers*	*Performing addition, subtraction, multiplication, and division on signed rational numbers*	*Performing addition, subtraction, multiplication, and division on signed rational numbers*
2	*Working with ratios and proportions*	*Finding the slope of a line*	*Evaluating algebraic expressions by substitution*
3	Working with percent (e.g., simple interest, tax, and markdowns)	*Using the Pythagorean theorem*	*Simplifying algebraic expressions*
4	Locating points in the coordinate plane†	*Solving linear equations* and inequalities *in one variable*	*Solving linear equations* and inequalities *in one variable*
5	*Simplifying algebraic expressions*	*Simplifying algebraic expressions*	Performing operations with integer exponents
6	Converting fractions to decimals and decimals to fractions†	*Evaluating algebraic expressions by substitution*	*Finding the slope of a line*
7	*Evaluating algebraic expressions by substitution*	*Working with ratios and proportions*	*Working with ratios and proportions*
8	Finding the mean*	Working with linear relationships	Working with number properties (e.g., divisibility, even/odd, and positive/negative)
9	*Solving linear equations in one variable**	Graphing linear equations in two variables	Performing addition, subtraction, and multiplication of polynomials
10	Computing the area and perimeter of polygons*	Using the quadratic formula	Computing a particular value of a function
11	*Finding the slope of a line*	Solving systems of two linear equations in two variables algebraically	Working with linear relationships
12	*Using the Pythagorean theorem*	Working with right triangle trigonometry	*Using the Pythagorean theorem*

*The skill is a subset of a skill listed on the high school and college faculty surveys.

†The skill was included on the middle school/junior high survey only.

Curriculum Consultants

ACT asked eight mathematics educators, representing various teaching levels and areas of the country, to respond with a short essay to each of five questions about the Mathematics survey results, the EPAS Mathematics Tests, and the mathematics curriculum. Table 3.5 identifies these eight consultants.

The consultants generally agreed that the survey results accorded with their sense of the curriculum. They further concurred that the results would provide helpful guidance to test development if interpreted with care, that is, with due allowance for the perspective of each of the respondent groups.

The EPAS Mathematics Tests are, in the consultants' opinions, well matched to school curricula and to college-level expectations. For example, the consultants found item sets (which present examinees with a problem situation and 2–4 items about it) and items couched in real-world contexts to be in line with the curriculum and to constitute an appropriate proportion of the tests.

The consultants identified several trends in the mathematics curriculum. One instance is increased emphasis on probability/statistics/data analysis topics. ACT will continue to monitor such changes and weigh the evidence to determine whether changes in the EPAS Mathematics Test specifications might be warranted.

At this time, ACT's analysis of the survey results, together with the consultants' consensus, supports the general approach and the specific test specifications of the EPAS Mathematics Tests.

Table 3.5
Mathematics Curriculum Consultants

Name	Title and affiliation
Ms. Cindy J. Boyd	Mathematics Teacher, Abilene High School, Abilene, Texas
Dr. Stan Chadick	Professor of Mathematics, Northwestern State University, Natchitoches, Louisiana
Dr. Luis Ortiz-Franco	Associate Professor, Department of Computer Science, Mathematics, and Physics, Chapman University, Orange, California
Mr. Dave Robathan	Mathematics Teacher, Shasta High School, Redding, California
Mr. David J. Tschiggfrie	Mathematics Teacher (recently retired), Washington Junior High School, Dubuque, Iowa Adjunct Professor of Mathematics, Northeast Iowa Community College
Dr. Zalman Usiskin	Professor of Education, University of Chicago, Chicago, Illinois Director, University of Chicago School Mathematics Project
Ms. Stacey Weinand	Mathematics and Science Partnership Coordinator, Oklahoma State Regents for Higher Education, Oklahoma City, Oklahoma
Dr. Rose Mary Zbiek	Associate Professor of Mathematics Education, Pennsylvania State University, University Park, Pennsylvania

Mathematics Test Specifications

The content areas for the EXPLORE, PLAN, and ACT Assessment Mathematics Tests are summarized in Table 3.6. Included in this table is the number (and proportion) of questions in each content area. As can be seen from the table, there is a clear progression in the content coverage of the tests from the 8th- to the 10th- to the 12th-grade-level programs.

Several points need to be made about the labeling of the content areas, especially at the 8th-grade level. At Grade 8, consistent with the National Council of Teachers of Mathematics (NCTM) Standards, "Basic Statistical/Probability Concepts" does not refer to the content of a formal statistics course, but to the ability to process data. Similarly, 8th-grade "Pre-Geometry" deals with use of figures and diagrams to solve mathematical problems. At levels higher than Grade 8, content definitions are consistent with standard course titles in high school.

The cognitive levels assessed by the Mathematics Tests are summarized in Table 3.7. The numbers (and proportions) of questions at each cognitive level are reported in this table. Although at first sight the increase in the proportion of "Knowledge and Skills" questions, and the decline in the proportion of "Understanding Concepts/Integrating Conceptual Understanding" questions, with increasing grade level may seem surprising, it must be remembered that at the higher grade levels the content areas are more challenging.

Table 3.6
Mathematics Test Specifications

	Testing program		
Content area	**EXPLORE**	**PLAN**	**ACT Assessment**
Basic Statistical/Probability Concepts	4 (.13)	*	*
Pre-Algebra	10 (.33)	14 (.35)	14 (.23)
Elementary Algebra	9 (.30)	8 (.20)	10 (.17)
Pre-Geometry	7 (.23)		
Plane Geometry		11 (.27)	14 (.23)
Coordinate Geometry		7 (.18)	9 (.15)
Intermediate Algebra			9 (.15)
Trigonometry			4 (.07)
Total	**30**	**40**	**60**

*On PLAN and the ACT Assessment, questions involving statistics/probability are included in the Pre-Algebra category.

Table 3.7
Cognitive Specifications for the Mathematics Tests

	Testing program		
Cognitive level	**EXPLORE**	**PLAN**	**ACT Assessment**
Knowledge and Skills	8 (.267)	14 (.350)	30 (.500)
Direct Application	8 (.267)	12 (.300)	17 (.283)
Understanding Concepts/Integrating Conceptual Understanding	14 (.467)	14 (.350)	13 (.217)
Total	**30**	**40**	**60**

Although a detailed description of the mathematics continuum is provided in the next section, it must be stressed that Mathematics Test forms are produced by sampling from the domains, rather than by testing every specific skill on every form. Students are advised to prepare for these tests by obtaining a thorough grounding in the full content domain rather than by trying to guess the specific content that will appear on a test form. Each form is a unique sample from the broad content domain; no particular topic in the content areas is guaranteed to appear on a given test form.

Mathematics Continuum Content and Cognitive Level Descriptions

Cognitive Levels

Knowledge and skills. Questions at this level require the student to use one or more facts, definitions, formulas, or procedures to solve problems that are presented in purely mathematical terms.

Direct application. Questions at this level require the student to use one or more facts, definitions, formulas, or procedures to solve straightforward problems set in real-world situations.

Understanding concepts. Questions at this level test the student's depth of understanding of major concepts by requiring reasoning from a concept to reach an inference or a conclusion.

Integrating conceptual understanding. Questions at this level test the student's ability to achieve an integrated understanding of two or more major concepts so as to solve nonroutine problems.

Content Areas

Basic Statistical/Probability Concepts. Questions in this content area (which is treated explicitly in EXPLORE, and implicitly as part of the Pre-Algebra content area in PLAN and the ACT Assessment) involve elementary counting and rudimentary probability; data collection, representation, and interpretation; reading and relating graphs, charts, and other representations of data; and other appropriate topics. All of these topics are addressed at a level preceding formal statistics. Questions in this content area cover the following topics:

- Counting and counting techniques
- The concept of probability
- Mean, median, and mode
- Data collection and representation
- Reading and interpreting graphs, charts, and other representations of data

Pre-Algebra. Questions in this content area are based (as appropriate for the grade levels across EXPLORE, PLAN, and the ACT Assessment) on basic operations using whole numbers, decimals, fractions, and integers; place value; square roots and approximations; the concept of exponents; scientific notation; factors; ratio, proportion, and percent; linear equations in one variable; absolute value and ordering numbers by value; elementary counting techniques and simple probability; data collection, representation, and interpretation; and understanding simple descriptive statistics. Questions in pre-algebra cover the following topics:

- Addition, subtraction, multiplication, and division of whole numbers, decimals, fractions, and integers
- Positive integer exponents
- Prime factorization
- Comparison of fractions
- Ratio and proportion
- Conversion of fractions to decimals, and conversion of decimals to fractions
- Absolute value
- Solution of linear equations in one variable (This is an Elementary Algebra topic for EXPLORE.)
- Percent
- Scientific notation
- Square roots and irrational numbers
- Operations with real numbers (field axioms)
- Order properties for real numbers
- Common factors and common multiples

Elementary Algebra. Questions in this content area are based (as appropriate for the grade levels across EXPLORE, PLAN, and the ACT Assessment) on properties of exponents and square roots, evaluation of algebraic expressions through substitution, using variables to express functional relationships, understanding algebraic operations, and the solution of quadratic equations by factoring. Questions in elementary algebra cover the following topics:

- Evaluation of algebraic expressions by substitution
- Simplification of algebraic expressions
- Addition, subtraction, and multiplication of polynomials
- Factorization of polynomials
- Solution of quadratic equations by factoring
- Formula manipulation and field properties of algebraic expressions

Pre-Geometry. Questions in this category (which applies to EXPLORE only) involve the use of scales and measurement systems, plane and solid geometric figures and associated relationships and concepts, the concept of angles and their measures, parallelism, relationships of triangles, properties of a circle, the Pythagorean theorem, and other appropriate topics. All of these topics are addressed at a level preceding formal geometry. Questions in pre-geometry cover the following topics:

- Using measurement systems
- Using rulers and other scales
- Concepts and relationships for plane and solid geometric figures
- Calculation of perimeter, area, and volume with formulas for selected geometric figures
- The concept of angle and angle measure
- Parallelism
- Properties of triangles
- Properties of circles
- Pythagorean theorem

Plane Geometry. Questions in this content area are based (as appropriate for the grade levels across PLAN and the ACT Assessment) on the properties and relations of plane figures, including angles and relations among perpendicular and parallel lines; properties of circles, triangles, rectangles, parallelograms, and trapezoids; transformations; the concept of proof and proof techniques; volume; and applications of geometry to three dimensions. Items in plane geometry cover the following topics:

- Identification of plane geometric figures
- Basic properties of a circle: radius, diameter, and circumference
- Measurement and construction of right, acute, and obtuse angles
- Parallel lines and transversals
- Congruent and similar triangles
- Areas of circles, triangles, rectangles, parallelograms, trapezoids, and, with formulas, other figures
- Pythagorean theorem
- Lines, segments, and rays
- Perpendicular lines
- Properties of triangles
- Ratio of sides in 45°-45°-90° triangles and 30°-60°-90° triangles
- Circumference and arc length

Coordinate Geometry. Questions in this content area are based (as appropriate for the grade levels across PLAN and the ACT Assessment) on graphing and the relations between equations and graphs, including points, lines, polynomials, circles, and other curves; graphing inequalities; slope; parallel and perpendicular lines; distance; midpoints; and conics. Questions in coordinate geometry cover the following topics:

- Graphing on the number line
- Identification and location of points in the coordinate plane
- Determination of graphs of functions and relations in the plane by plotting points
- Graphs of linear equations in two variables
- Slope of a line
- Distance formula for points in the plane

Intermediate Algebra. Questions in this content area (which applies to the ACT Assessment only) are based on an understanding of the quadratic formula, rational and radical expressions, absolute value equations and inequalities, sequences and patterns, systems of equations, quadratic inequalities, functions, modeling, matrices, roots of polynomials, and complex numbers. Questions in intermediate algebra cover the following topics:

- Solution of linear inequalities in one variable
- Operations with integer exponents
- Operations with rational expressions
- Slope-intercept form of a linear equation
- Operations with radical expressions
- Quadratic formula
- Graphs of parabolas, circles, ellipses, and hyperbolas
- Zeros of polynomials
- Rational exponents
- Equations of circles
- Solution of systems of two linear equations in two variables
- Simple absolute value equations and inequalities
- Graphical solutions to systems of equations and/or inequalities
- Equations of parallel and perpendicular lines

Trigonometry. Questions in this content area (which applies to the ACT Assessment only) are based on understanding trigonometric relations in right triangles; values and properties of trigonometric functions; graphing trigonometric functions; modeling using trigonometric functions; use of trigonometric identities; and solving trigonometric equations. Questions in trigonometry cover the following topics:

- Right triangle trigonometry
- Trigonometric functions
- Graphs of trigonometric functions, including amplitude, period, and phase shift
- Trigonometric identities
- Addition formulas for sine and cosine
- Simple trigonometric equations

Table 3.8

Mathematics Curriculum Survey Results: Calculators in the Classroom, by Academic Level

	Middle school/ junior high*	High school*	College
Use of calculators in mathematics courses			
Required	39%	67%	49%
Optional	57%	39%	41%
Not permitted	26%	6%	10%
Type of calculator recommended or required, as a percentage of respondents who indicated calculators are required or optional in one of their classes			
Basic (4-function)	33%	8%	7%
Scientific	61%	52%	29%
Graphing	28%	71%	54%
Statistical	N/A†	N/A†	3%
Symbolic Manipulator	1%	3%	3%

*Percentages for middle school/junior high and high school may not total 100% because respondents were allowed to check all that applied.

†Middle school/junior high respondents were not given this option.

Table 3.9

Mathematics Curriculum Survey Results: Percent Taught, Mean Rating, and Rank for Each Process Skill, by Respondent Group

	Middle school/junior high				High school				College	
Process skill	% Taught before Grade 8	% Taught before Grade 9	Mean importance rating	Rank	% Taught before Grade 10	% Taught before Grade 12	Mean importance rating	Rank	Mean Importance rating	Rank
Performing basic operations with a calculator	83	89	4.33	11	95	98	4.56	3	3.90	1
Recalling quickly basic facts, definitions, formulas, and algebraic procedures then using them correctly to solve a problem	62	87	4.43	10	79	97	4.47	5	3.79	2
Planning and carrying out a strategy for solving multistep problems	81	94	4.63	3	85	96	4.63	1	3.74	3
Solving problems posed in real-world settings and interpreting the solution	86	93	4.67	1	86	96	4.61	2	3.70	4
Recognizing and using patterns to solve problems	84	93	4.58	4	80	95	4.41	8	3.69	5
Choosing an appropriate method for calculating (e.g., mental, paper and pencil, calculator, or estimation)	87	93	4.49	7	76	86	4.42	7	3.58	6
Applying mathematical ideas to new contexts	74	88	4.51	5	73	93	4.42	6	3.56	7
Reading and interpreting graphs, charts, and other data representations	88	94	4.65	2	87	96	4.52	4	3.55	8
Recognizing generalizations of mathematical ideas	73	88	4.48	9	74	92	4.33	10	3.49	9
Recognizing when essential data are missing	81	92	4.51	6	78	88	4.36	9	3.20	10
Using estimation to approximate solutions	89	95	4.48	8	80	90	4.33	11	3.19	11
Applying theorems to solve a problem	12	38	4.02	15	19	97	4.12	14	2.94	12
Solving several problems representing different aspects/ components of a larger problem or scenario	46	75	4.23	12	52	84	4.15	13	2.93	13
Formulating new patterns or structures	57	79	4.10	14	57	85	3.97	15	2.86	14
Using the graphical capabilities of a calculator	10	43	3.80	17	42	88	4.19	12	2.82	15
Recalling theorems and more complex formulas when needed to solve a problem	13	42	3.91	16	18	94	3.92	16	2.81	16
Demonstrating concepts using manipulatives and/or pictorial representations	86	93	4.10	13	80	89	3.85	17	2.75	17
Constructing and/or critiquing proofs, either informal or formal	4	11	3.44	20	10	95	3.51	19	1.82	18
Using the symbolic manipulation capabilities of a calculator	10	29	3.47	19	17	51	3.28	20	1.79	19
Using the statistical capabilities of a calculator	15	39	3.71	18	20	66	3.66	18	1.70	20
Process skills median rating			4.38				4.33		3.20	

Table 3.10

Mathematics Curriculum Survey Results: Percent Taught, Mean Rating, and Rank for Each Content Skill, by Respondent Group

	Middle school/junior high					
Content skill	% Taught before Grade 8	% Taught before Grade 9	% Not taught at all	Mean importance rating	SD	Rank
Performing addition, subtraction, multiplication, and division on signed rational numbers	81	92	0	4.74	0.69	1
Working with scientific notation	75	91	0	3.58	1.15	63
Converting fractions to decimals and decimals to fractions	92	95	0	4.54	0.76	7
Computing common factors and common multiples of numbers	91	95	0	4.28	0.91	24
Computing the absolute value of a number	67	93	0	3.66	1.13	59
Ordering real numbers, including ordering on the number line	84	94	0	4.30	0.97	18
Working with ratios and proportions	82	94	0	4.59	0.66	3
Working with percent (e.g., simple interest, tax, and markdowns)	78	94	0	4.56	0.78	4
Working with number properties (e.g., divisibility, even/odd, and positive/negative)	88	93	0	4.30	0.91	20
Evaluating algebraic expressions by substitution	68	90	0	4.50	0.79	9
Simplifying algebraic expressions	61	89	0	4.55	0.75	5
Solving linear equations in one variable	62	90	0	4.60	0.70	2
Solving linear inequalities in one variable	43	87	0	4.33	0.85	16
Solving absolute value equations and inequalities	20	69	3	3.88	1.03	54
Raising numbers to whole number powers	73	93	1	4.38	0.81	13
Performing operations with integer exponents	33	80	0	4.21	0.94	30
Working with rational exponents	21	58	10	3.86	1.08	55
Simplifying radicals	15	53	4	3.95	1.05	50
Performing addition, subtraction, and multiplication of polynomials	12	55	2	4.27	0.87	26
Solving quadratic equations by factoring	4	40	5	4.18	0.99	31
Using the quadratic formula	2	35	8	4.11	1.02	39
Using the discriminant	1	19	30	3.51	1.22	64
Working with graphs of quadratic equations and functions	5	34	10	4.00	1.04	47
Approximating roots of polynomial and rational equations from graphs	2	24	26	3.68	1.13	58
Determining roots of polynomial and rational equations algebraically	1	27	16	3.90	1.08	52
Computing a particular value of a function	20	60	8	4.14	0.96	37
Recognizing relationships between a function or an equation and its graph	12	58	6	4.14	0.98	35
Performing matrix addition and multiplication	8	31	28	3.34	1.23	65
Solving systems of two linear equations in two variables algebraically	4	46	4	4.23	0.96	28

Table 3.10

Mathematics Curriculum Survey Results: Percent Taught, Mean Rating, and Rank for Each Content Skill, by Respondent Group *(continued)*

	Middle school/junior high					
Content skill	% Taught before Grade 8	% Taught before Grade 9	% Not taught at all	Mean importance rating	SD	Rank
Solving systems of equations and inequalities graphically	5	49	4	4.13	1.00	38
Working with series and sequences (e.g., arithmetic and geometric)	59	77	10	4.04	1.00	45
Graphing on the number line	81	92	0	4.28	0.93	25
Graphing solutions to inequalities on the number line	53	92	0	4.08	0.96	42
Locating points in the coordinate plane	85	93	0	4.55	0.74	6
Working with transformations graphically	51	74	12	3.70	1.15	57
Working with transformations algebraically	12	44	22	3.59	1.22	62
Graphing linear equations in two variables	15	64	3	4.31	0.88	17
Finding the slope of a line	18	75	0	4.41	0.80	11
Working with equations of parallel and perpendicular lines	8	55	3	4.17	0.93	34
Working with linear relationships	15	66	3	4.30	0.85	19
Using the midpoint formula	5	41	9	3.89	1.05	53
Using the distance formula	20	55	7	4.10	1.02	40
Using the Pythagorean theorem	39	83	1	4.40	0.87	12
Working with right, acute, and obtuse angles and their properties	73	88	2	4.29	0.88	21
Computing an unknown angle measure in polygons	48	82	5	4.05	0.95	44
Working with lines, segments, and rays and their properties	72	87	3	4.14	0.91	36
Working with parallel lines, transversals, and angle measures	54	83	4	4.18	0.90	32
Classifying plane geometric figures	74	87	2	4.17	0.94	33
Working with properties of special quadrilaterals	53	73	9	3.98	1.00	48
Working with side length relationships in 45°-45°-90° triangles and 30°-60°-90° triangles	19	54	14	4.02	1.01	46
Working with similar triangles	61	86	3	4.09	0.94	41
Working with congruent triangles	59	83	5	4.08	0.93	43
Working with circles (e.g., radius, diameter, arc, and chord)	69	86	4	4.29	0.80	22
Working with inscribed and circumscribed polygons and circles	20	39	25	3.62	1.13	61
Computing the area and circumference of circles	75	91	2	4.37	0.86	14
Computing the area and perimeter of polygons	82	93	2	4.45	0.82	10
Computing volume of prisms and cylinders	59	89	5	4.22	0.91	29
Working with right triangle trigonometry	11	41	23	3.96	1.06	49

Table 3.10

Mathematics Curriculum Survey Results: Percent Taught, Mean Rating, and Rank for Each Content Skill, by Respondent Group *(continued)*

	Middle school/junior high					
Content skill	% Taught before Grade 8	% Taught before Grade 9	% Not taught at all	Mean importance rating	SD	Rank
Representing data (e.g., circle graphs, scatterplots, and frequency distributions)	71	92	1	4.29	0.92	23
Finding the mean	87	93	0	4.50	0.79	8
Finding the median and mode	86	94	0	4.37	0.85	15
Computing the probability of a simple event	75	90	2	4.23	0.91	27
Using counting techniques	71	84	8	3.95	1.04	51
Working with Venn diagrams	66	84	10	3.74	1.09	56
Working with mutually exclusive, dependent, and independent events	35	71	11	3.64	1.09	60
Content skills median rating				4.17		

Table 3.10

Mathematics Curriculum Survey Results: Percent Taught, Mean Rating, and Rank for Each Content Skill, by Respondent Group *(continued)*

	High school						College		
Content skill	% Taught before Grade 10	% Taught before Grade 12	% Not taught at all	Mean importance rating	SD	Rank	Mean importance rating	SD	Rank
Performing addition, subtraction, multiplication, and division on signed rational numbers	97	99	1	4.77	0.59	1	4.71	0.72	1
Working with ratios and proportions	95	99	0	4.59	0.71	7	4.09	1.10	7
Working with percent (e.g., simple interest, tax, and markdowns)	95	98	2	4.39	0.90	20	3.66	1.27	15
Working with number properties (e.g., divisibility, even/odd, and positive/negative)	96	97	2	4.33	0.93	23	4.05	1.18	8
Evaluating algebraic expressions by substitution	93	99	0	4.61	0.68	6	4.42	0.93	2
Simplifying algebraic expressions	95	99	0	4.68	0.63	5	4.33	1.04	3
Solving linear equations and inequalities in one variable	94	99	0	4.73	0.58	4	4.19	1.15	4
Solving absolute value equations and inequalities	68	98	0	3.99	0.93	42	3.32	1.36	23
Performing operations with integer exponents	82	98	0	4.45	0.78	15	4.14	1.19	5
Working with rational exponents	33	97	0	4.15	0.92	35	3.65	1.43	16
Performing operations on radical expressions/equations	45	97	0	4.20	0.89	30	3.48	1.39	20
Performing addition, subtraction, and multiplication of polynomials	79	98	0	4.46	0.79	14	3.89	1.38	9
Performing polynomial long division	36	92	3	3.37	1.17	71	2.66	1.36	31
Solving quadratic equations by factoring	69	97	0	4.40	0.87	18	3.64	1.46	17
Using the quadratic formula	53	98	0	4.53	0.72	10	3.59	1.45	18
Using the discriminant	21	92	2	3.51	1.09	60	2.49	1.29	39
Solving quadratic inequalities	12	91	2	3.50	1.03	64	2.59	1.40	35
Working with graphs of quadratic equations and functions	30	96	0	4.26	0.84	27	3.34	1.53	22
Approximating roots of polynomial and rational equations from graphs	15	91	2	3.84	1.02	47	2.61	1.46	34
Determining roots of polynomial and rational equations algebraically	17	92	1	4.07	0.93	39	2.79	1.45	29
Performing operations with rational expressions	49	96	0	4.09	0.93	38	3.45	1.43	21
Implementing remainder and factor theorems	7	82	4	3.37	1.04	72	2.17	1.22	51
Computing a particular value of a function	57	97	0	4.44	0.80	17	3.81	1.41	10
Recognizing relationships between a function or an equation and its graph	44	96	0	4.39	0.81	19	3.70	1.46	14
Working with functions (e.g., inverses, composition, and range and domain)	27	94	0	4.12	0.87	36	3.27	1.56	24
Working with graphs of rational functions	17	86	2	3.87	0.95	46	2.79	1.50	29
Working with logarithmic and exponential functions	2	85	2	3.92	0.96	45	2.95	1.59	27

Table 3.10

Mathematics Curriculum Survey Results: Percent Taught, Mean Rating, and Rank for Each Content Skill, by Respondent Group *(continued)*

	High school						College		
Content skill	% Taught before Grade 10	% Taught before Grade 12	% Not taught at all	Mean importance rating	SD	Rank	Mean importance rating	SD	Rank
Working with logic statements (e.g., converse, contrapositive, and if-then)	11	90	5	3.36	1.04	73	2.03	1.19	57
Performing matrix addition and multiplication	18	85	5	3.36	1.05	74	1.55	1.04	79
Finding determinants	7	84	5	3.29	1.02	79	1.54	0.99	80
Solving systems of two linear equations in two variables algebraically	69	98	0	4.48	0.74	11	2.99	1.45	26
Solving systems of equations and inequalities graphically	57	98	1	4.10	0.89	37	2.58	1.38	36
Working with series and sequences (e.g., arithmetic and geometric)	15	80	6	3.46	1.01	65	1.86	1.24	61
Working with sequences that are defined recursively	4	65	16	3.11	1.11	80	1.68	1.14	72
Computing the sum of an infinite geometric series	2	59	10	3.09	1.06	81	1.64	1.16	73
Working with sigma notation	1	55	10	3.31	1.06	77	2.11	1.41	54
Working with sets and set notation	44	76	13	3.42	1.15	68	2.48	1.31	40
Working with equations of parabolas, circles, ellipses, and hyperbolas	2	85	2	3.72	0.94	51	2.47	1.40	41
Graphing parabolas, circles, ellipses, and hyperbolas	2	84	2	3.68	0.98	53	2.45	1.39	42
Determining a locus of points	2	68	18	3.06	1.17	82	1.77	1.16	66
Working with vectors in a plane	3	53	14	3.32	1.11	76	1.46	0.91	83
Working with parametric equations	2	35	23	3.01	1.16	83	1.53	1.03	81
Working with transformations graphically	16	81	5	3.54	1.05	58	2.04	1.31	56
Working with transformations algebraically	12	75	8	3.50	1.10	63	2.06	1.31	55
Working with discrete and continuous graphs	9	55	12	3.41	1.06	69	2.32	1.37	47
Graphing linear equations in two variables	80	98	0	4.55	0.73	9	3.71	1.43	12
Finding the slope of a line	91	99	0	4.74	0.58	2	4.13	1.25	6
Working with equations of parallel and perpendicular lines	79	98	0	4.45	0.78	16	3.51	1.43	19
Working with linear relationships	82	99	0	4.57	0.68	8	3.80	1.32	11
Using the midpoint formula	55	98	0	4.17	0.93	34	2.83	1.39	28
Using the distance formula	50	97	0	4.35	0.85	22	3.22	1.47	25
Using the Pythagorean theorem	70	98	0	4.74	0.55	3	3.71	1.42	12
Applying properties of right, acute, and obtuse angles	32	99	0	4.32	0.81	25	2.63	1.44	33
Applying properties of lines, segments, and rays	30	99	0	4.20	0.87	31	2.34	1.37	44
Working with parallel lines, transversals, and angle measures	24	99	0	4.23	0.85	29	2.26	1.35	48
Working with properties of special quadrilaterals	19	99	0	4.02	0.89	40	1.83	1.12	62

Table 3.10

Mathematics Curriculum Survey Results: Percent Taught, Mean Rating, and Rank for Each Content Skill, by Respondent Group *(continued)*

	High school						College		
Content skill	% Taught before Grade 10	% Taught before Grade 12	% Not taught at all	Mean importance rating	SD	Rank	Mean importance rating	SD	Rank
Working with side length relationships in 45°-45°-90° triangles and 30°-60°-90° triangles	14	97	0	4.31	0.83	26	2.41	1.47	43
Working with congruent and similar triangles	24	99	0	4.26	0.83	28	2.33	1.33	45
Working with circles (e.g., radius, diameter, arc, and chord)	27	98	1	4.20	0.88	32	2.58	1.41	36
Working with inscribed and circumscribed polygons and circles	13	97	2	3.60	1.02	56	1.64	0.98	73
Computing the area and perimeter of polygons and circles	48	98	0	4.47	0.74	13	2.65	1.38	32
Computing volume (e.g., prisms, pyramids, and cylinders)	41	98	1	4.33	0.83	24	2.33	1.36	45
Working with right triangle trigonometry	16	94	0	4.47	0.75	12	2.57	1.64	38
Using trigonometric identities	3	70	1	3.84	1.02	48	2.26	1.55	48
Solving trigonometric equations	3	67	2	3.96	0.97	43	2.14	1.45	52
Using the law of sines and law of cosines	2	73	2	3.96	0.97	44	1.94	1.34	59
Working with graphs of trigonometric functions, including amplitude, period, and phase shift	0	61	2	3.81	0.99	49	2.20	1.56	50
Representing data (e.g., circle graphs, scatterplots, and frequency distributions)	59	88	4	4.18	0.99	33	2.12	1.37	53
Determining a line of best-fit by eye for a set of data	51	86	7	3.75	1.15	50	1.81	1.20	64
Working with correlation	28	72	15	3.52	1.19	59	1.59	1.07	76
Finding the mean, median, and mode	86	96	1	4.38	0.88	21	1.83	1.29	62
Finding the variance and standard deviation of data	10	57	21	3.42	1.18	67	1.56	1.11	78
Working with the normal distribution	8	52	25	3.36	1.18	75	1.49	1.05	82
Computing the probability of a simple event	65	90	4	4.01	1.03	41	1.70	1.26	69
Using counting techniques	51	81	10	3.69	1.15	52	1.70	1.21	69
Working with Venn diagrams	53	82	14	3.31	1.16	78	1.73	1.16	68
Working with mutually exclusive, dependent, and independent events	26	73	9	3.38	1.07	70	1.57	1.13	77
Working with combinations, permutations, and the binomial theorem	8	68	8	3.44	1.04	66	1.60	1.11	75
Finding derivatives	1	15	15	3.64	1.23	54	1.75	1.41	67
Finding definite integrals	0	12	20	3.55	1.27	57	1.70	1.37	69
Determining maxima, minima, and points of inflection for functions	1	33	9	3.60	1.13	55	1.90	1.42	60
Understanding continuity	0	29	11	3.51	1.13	61	2.00	1.43	58
Finding the limit of an expression	0	20	10	3.51	1.15	62	1.81	1.38	64
Content skills median rating				3.99			2.45		