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The Effect of Item Response Dependency on Trait or Ability Dimensionality

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ABSTRACT

The purpose of this paper was to investigate various levels of item response dependency using principal component analyses. Item response data were simulated using an IRT-based dependency model which describes a two-state Markov process. Results indicated that when the IRT assumption of local independence was violated, items within a dependent sequence were clearly identified by their loadings on a second principal component, in addition to the common first principal component shared by all of the items. Under "realistic conditions" of local dependence, the response data retained their unidimensional characteristics. Concern for the effect that item dependency may have on ability estimation is also discussed.

The Effect of Item Response Dependency on Trait or Ability Dimensionality

The assumption of local independence allows the joint probability distribution of observing a response vector, $\underset{\sim}{U}' = (u_1 u_2 \ldots u_k)$ given ability θ_i to be written as a product of $\underline{k}$ marginal probability functions or

$$P(U_1 = u_1, U_2 = u_2, \ldots, U_k = u_k | \theta_i) = \prod_{j=1}^{k} [P(U_j = u_j | \theta_i)] . \tag{1}$$

A particular violation of this assumption can arise in the following way. Suppose that, for a $\underline{k}$-item test, $\underline{m}$ of the items form a subset such that

$$P(U_1 = u_1, U_2 = u_2, \ldots, U_m = u_m, U_{m+1} = u_{m+1}, \ldots, U_k = u_k | \theta_i) =$$

$$P(U_1 = u_1 | \theta_i) P(U_2 = u_2 | \theta_i, u_1) \ldots P(U_m = u_m | \theta_i, u_{m-1}) \prod_{j=m+1}^{k} [P(U_j = u_j | \theta_i)] . \tag{2}$$

For example, the three geometry items pictured in Figure 1 could represent the first three items on a 20-item geometry test. The joint probability density function for an examinee with ability θ_i could be written as

$$P(U_1 = u_1, U_2 = u_2, U_3 = u_3, \ldots U_{20} = u_{20} | \theta_i) =$$

$$P(U_1 = u_1 | \theta_i) P(U_2 = u_2 | \theta_i, u_1) P(U_3 | \theta_i, u_2) \prod_{j=4}^{20} [P(U_j = u_j | \theta_i)]$$

in order to account for item response dependence between items 1 and 2 and between items 2 and 3.

Does the joint probability density in equation (2) imply that the dimensionality of the space defined by the item responses as greater than one, even though only a scalar value of θ_i is assumed? To investigate this question, we have used a finite, two-state (0 or 1, incorrect or correct) Markov chain or process to model the dependence within the m-item sequence.

Let $P_j(\theta_i)$ represent the probability of an examinee with trait measure θ_i answering test item j correctly, independently of any other test item. Then define a transition matrix between any adjacent items, j-1 and j in the k-item test as specified below.

$$
\begin{array}{cc|c|c|}
 & & \multicolumn{2}{c}{j\text{th item}} \\
 & & 0 & 1 \\
\cline{3-4}
 & 0 & 1-\alpha^*_{ij} & \alpha^*_{ij} \\
\cline{3-4}
j\text{th-1 item} & & & \\
 & 1 & \beta^*_{ij} & 1-\beta^*_{ij} \\
\cline{3-4}
\end{array}
$$

In this model, α^*_{ij} represents the probability that an examinee with trait θ_i will move from an incorrect response on item j-1 (state 0) to a correct response on item j (state 1). Similarly, β^*_{ij} represents a transition probability from a correct response on item j-1 to an incorrect response on item j. The probabilities, $1 - \alpha^*_{ij}$ and $1 - \beta^*_{ij}$ imply state consistency between items.

We note that items j and j-1 are assumed to be adjacent test items only for the purpose of discussion in this paper. This is not a requirement, however, and in fact all discussion may be generalized to any two test items, j and $j-\tau$, where $\tau = 1,2,\ldots,k-1$ and $j = \tau+1,\tau+2,\ldots,m$.

These four cell probabilities are functions of (1) the jth item-by-ith person interaction, as given by $P_j(\theta_i)$, and (2) the amount and direction of any item dependency. This definition of the transition probabilities is similar in structure to the latent Markov chain model described by Lazarsfeld and Henry (1968). These probabilities are defined as follows.

$$\alpha^*_{ij} = \alpha P_j(\theta_i)$$

and

$$\beta^*_{ij} = \beta Q_j(\theta_i)$$

where

$$Q_j(\theta_i) = 1 - P_j(\theta_i) .$$

The parameters, α and β, are weights that describe the dependency relationship with $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 1$. For the purpose of the simple examples provided in this paper, the weights are assumed to be constant but not necessarily equal to each other for all adjacent pairs of items in the m-item sequence. This is not required, however; within one m-item sequence, α and β may take on any of the values in the range described above between any two pairs of items.

Once the transition probabilities have been defined, the complement probabilities can be written as

$$1 - \alpha^*_{ij} = 1 - \alpha P_j(\theta_i)$$

and

$$1 - \beta^*_{ij} = 1 - \beta Q_j(\theta_i) \quad .$$

The dependency weights, α and β, fix the amount of dependency among the *m* items, and since they can be assigned values independently of one another, they also fix the direction of item dependency (e.g., from correct to incorrect). When $\alpha = \beta = 1$, the items are independent, and when $\alpha = \beta = 0$, the items are completely dependent. This is more easily seen from the definition of the success probability for item *j* that results from the item dependency on previous items, or $P'_j(\theta_i)$. $P'_j(\theta_i)$ is the probability of answering the *j*th item correctly, given an incorrect response to the previous item *or* given a correct response to the previous item. In other words,

$$\begin{aligned} P'_j(\theta_i) &= Q'_{j-1}(\theta_i)\alpha^*_{ij} + P'_{j-1}(\theta_i)\{1 - \beta^*_{ij}\} \\ &= Q'_{j-1}(\theta_i)\alpha P_j(\theta_i) + P'_{j-1}(\theta_i)\{1 - \beta\, Q_j(\theta_i)\} \quad . \end{aligned} \tag{3}$$

When $\alpha = \beta = 1$,

$$\begin{aligned} P'_j(\theta_i) &= Q'_{j-1}(\theta_i)P_j(\theta_i) + P'_{j-1}(\theta_i)\{1 - Q_j(\theta_i)\} \\ &= \{1 - P'_{j-1}(\theta_i)\}P_j(\theta_i) + P'_{j-1}(\theta_i)P_j(\theta_i) \\ &= P_j(\theta_i) - P_j(\theta_i)P'_{j-1}(\theta_i) + P'_{j-1}(\theta_i)P_j(\theta_i) \\ &= P_j(\theta_i) \quad . \end{aligned}$$

This implies that the response for the *j*th item, given θ_i, depends only on the *j*th item's ICC.

Similarly, when $\alpha = \beta = 0$, $P_j(\theta_i) = P'_{j-1}(\theta_i)$ and the *j*th item response is solely determined by the previous item probability of a correct response. That is, no characteristics of the *j*th item have any influence on the correct response for item *j*.

A Measure of Item Dependence

The seriousness of the effect of the violation of local independence depends upon several factors and the interactions of these factors. These include the departure of α and β from 1.0, the item characteristics (i.e., difficulty and discrimination) of those items within the dependent sequence, the order of the items (e.g., easy-to-difficulty, difficult-to-easy), and the length of the dependent sequence, *m*.

One method of evaluating the severity of the violation of local independence within a given sequence of *m* items is to compute the sum of the absolute differences between the likelihood of each of the possible 2^m response patterns occurring under a joint density function of the Markov process or $P(U_1 = u_1|\theta_i)P(U_2 = u_2|\theta_i,u_1)\ldots P(U_m = u_m|\theta_i,u_{m-1})$ and the local independence of these *m* items or $\prod_{j=1}^{m} [P(U_j = u_j|\theta_i)]$.

When the differences are evaluated at some value of $\theta = \theta_0$ that is thought to be representative of the examinee population and summed over all possible 2^m response patterns, a measure of the departure from local independence can be computed. We have defined this value as a measure, Φ, where

$$\Phi = \sum_{\ell=1}^{2^m} |P'(\underset{\sim}{U}_\ell = \underset{\sim}{u}_\ell | \theta_0) - P(\underset{\sim}{U}_\ell = \underset{\sim}{u}_\ell | \theta_0)| \ , \tag{4}$$

$$P'(\underset{\sim}{U}_\ell = \underset{\sim}{u}_\ell | \theta_0) = P'(U_1 = u_1, U_2 = u_2, \ldots, U_m = u_m | \theta_0)$$

$$= P(U_1 = u_1 | \theta_0) P(U_2 = u_2 | \theta_0, u_1) \ldots P(U_m = u_m | \theta_0, u_{m-1}) \ ,$$

and

$$P(\underset{\sim}{U}_\ell = \underset{\sim}{u}_\ell | \theta_0) = P(U_1 = u_1, U_2 = u_2, \ldots, U_m = u_m | \theta_0)$$

$$= \prod_{j=1}^{m} \left[P(U_j = u_j | \theta_0)\right] \ .$$

It can be shown that, for any given ability value, θ_0, $0.0 \leq \Phi < 2.0$, regardless of how $P_j(\theta_0)$ is defined (e.g., one-, two-, or three-parameter logistic function). Obviously, when $\Phi = 0.0$, local independence holds throughout the m-item sequence (for all values of θ as well as $\theta = \theta_0$). As Φ approached 2.0, the degree to which local independence has been violated increases. Recall that Φ is a function of α and β, item characteristiics, item order, and the length of the dependent sequence, m.

Simulated Data Sets

The purpose of this investigation was to determine the effect of the violation of the assumption of local independence, as measured by Φ, on the dimensionality of the item response space and to study the trait estimates obtained under these circumstances. Twelve dāta sets were generated to simulate 1000 examinees' responses to a 50-item test. The examinees' ability distribution was assumed to be unidimensional with $\theta \sim N(0,1)$. Two of the data sets assumed local independence and differed only on the distribution of item difficulty. All sets assumed that $P_j(\theta_i)$ was a one-parameter logistic function of θ_i. Data set #1 allowed the difficulty or $\underline{b}$ parameter to be normally distributed with mean, 0 and standard deivation, 1. Data set #2 defined $\underline{b}$ to be uniformly distributed on the interval, [-.25, .25].

The inclusion of independent data sets facilitated a comparison and interpretation of the results of subsequent principal component analyses of phi coefficients on the remaining 10 dependent data sets. A one-parameter logistic function of θ was chosen to minimize the effect of nonlinearity of the item responses on ability. Furthermore, the two separate independent data sets mentioned previously were compared to see if a restriction of the 50-item difficulty range might insure a "pure," one-dimensional solution of the principal component analyses.

Table 1 gives the results of these principal component analyses in terms of the sizes of the first four eigenvalues. These results showed that restricting the item difficulty range, relative to the distribution of the abilities, eliminated the "second factor" or component that usually appears in principal component or factor analyses of item response data. Subsequently, all dependent data sets except one were generated with $\underline{b} \sim U(-.25,.25)$.

The ten dependent data sets ranged in the severity of the violation of the assumption of location independence. With the various values of α, β and $\underline{m}$ given in Table 1, the value of Φ was computed for $\theta = \theta_0 = .00$. Data set #'s 3 and 4 were considered to be cases of mild dependency; data set #'s 6 and 7 were considered to be cases of medium dependency; and set #'s 9 and 10 were considered strong cases. Three data sets (#'s 5, 8 and 11) combined previous effects to include two dependent sequences, as given in Table 1. Data set #12 was similar to #7 except that item difficulty was assumed to be normally distributed, or $\underline{b} \sim N(0,1)$.

Results of the Principal Component Analyses: Independent Data Sets

Table 1 shows that the independent data sets (#'s 1, and 2) gave different results in terms of the sizes of the first two eigenvalues. With the increased item difficulty variability in the first set, an expected difficulty or nonlinearity component, as reported by others (e.g., McDonald & Ahlawat, 1974), was clearly evident. As seen in Table 2, items which had either very high or very low values of $\underline{b}$ loaded on this second component, C_2. We have provided these results in order to (1) compare them to the results from data set #2 to show that we were able to eliminate or at least minimize the effect of the ICC curvature on the emergence of other factors and to (2) compare these results to those obtained when dependent sequences were embedded within the data sets.

By distributing the $\underline{b}$ parameter as $\underline{b} \sim U(-.25,.25)$, two things were accomplished. First, the importance of the second component due to the nonlinearity of $P_j(\theta_i)$ on θ was minimized. Secondly, the curvilinear relationship between item difficulty and the loadings on the first principal component (i.e., the relationship between item difficulty and the point biserial correlation coefficient) was eliminated. This made it easier to interpret the effects of item response dependency in data sets #3 - #11.

Results of the Principal Component Analyses: Dependent Data Sets

Table 1 shows that for mild or medium cases of dependency (i.e., $\Phi \leq 1.12$) a much weaker second component emerged. In fact for all intents and purposes, these cases produced fairly unidimensional solutions. For those sequences where strong item dependency was present, a stronger second component emerged.

Tables 3, 4 and 5 give the component loadings for these remaining data sets. (Boldface loadings are larger, in absolute value, than .250). One of the first observations to be made when comparing these tables to Table 2 and the loadings on C_1 from data set #1, is the size of these loadings. In the dependent data sets, these loadings remained large or even increased in magnitude as Φ increased. Table 6 gives the C_1 loadings for items, as they appeared in data set #2, that subsequently appeared in a dependent sequence. Most of the C_1 loadings increased as the degree of dependency increased.

The pattern of loadings on other factors when $\Phi \neq 0$ may be easiest to observe and describe in the cases when item dependence is greater than the mild cases (See Tables 4 and 5). In Table 4, for single sequences of dependency, a "dependency" component, although weak in the sense of the value of λ_2, did produce substantially large component loadings on C_2 for those items included in the sequence.

These loadings appear to have increased roughly in magnitude as the item appeared further from the start of the _m_-item sequence. When two sequences were included, as in data set #8, Table 4, two such "dependency components" emerged, although again we must state that λ_2 and λ_3 values were not large. Still, all other items not in the sequences did not load on these components with any magnitude close to .250, and most loaded less than .10. This indicated that these components were associated primarily with the dependency effect.

The same patterns of the loadings held in cases of stronger item response dependency, although the magnitudes of the loadings on all factors were larger. These can be seen in Table 5. The identical loadings for data set #10 were due to the fact that $\alpha = \beta = 0$; hence, there was complete dependency (i.e., only two response vectors were possible: (000000) or (111111)). The same thing occurred in data set #11. Although situations where $\alpha = \beta = 0$ are unrealistic, these conditions do provide maximum dependency in simulation work.

A final data set (#12) was included to show that this dependency component would remain distinct even when item difficulty varied. For this data set, $\underline{b} \sim N(0,1)$ as in set #1. The dependency parameters, α, β and $\underline{m}$ (and consequently Φ) were defined as in set #7. Table 7 shows the results. The items in the dependent sequency continued to load on a dependency component (C_3) while items with extreme $\underline{b}$ values loaded once again on C_2. Two of the items within the dependent sequence (#2 and #3) also loaded strongly on C_2, even though their $\underline{b}$ values were not really extreme. However, the dependency on the first item, a very difficult one, did make the subsequent items in the sequence more difficult. (For other examples of the effect of item difficulty and order in dependent item sequences, see Ackerman and Spray, 1986).

Estimates of θ for Dependent Sets

The second part of this investigation concerned the estimation of θ under situations where the assumption of local independence did not hold. Ability estimates werre obtained via the computer program, LOGIST 5 (Wingersky, Barton & Lord, 1982) for fixed values of item parameters (i.e., $\underline{a} = 1.0$, $\underline{c} = 0.0$ and $\underline{b}_j$ were 50 realizations from $\underline{b} \sim U(-.25,.25)$ as previously described). Table 8 summarizes the estimation results in terms of bias statistics. Data set #2, again was the independent set. The θ_i values used to generate the data were, of course, known and held constant throughout the simulations.

Table 8 shows that, in general LOGIST 5 underestimated θ when the item responses violated the assumption of local independence. In addition, as the strength of the item response dependency increased, the maximum amount of underestimation as well as overestimation increased, as did the variability of the estimates. The largest errors in estimation were not necessarily at the extreme values of θ, but were scattered rather uniformly throughout the θ range. A comparison of Figures 2 and 3, plots of $\hat{\theta}$ versus θ for data sets #2 and #11 respectively, show this increase in underestimation and variance.

Summary and Conclusions

When the local independence assumption is violated, the items within the dependent sequency load on a second principal component, in addition to the common first principal component shared by all of the items. It was discovered that, under realistic conditions normally encountered (i.e., where the length of the sequence, $\underline{m}$ would be small and the degree of the departure of α and β from 1.0 would not be severe), the response data still appeared to have retained their assumed unidimensional characteristics.

However, the fact that only those items within a given dependent sequence loaded significantly on a common component or components may offer a promising method of discovering item dependencies. These loadings can be fairly large in magnitude, even in cases of mild dependency, and when coupled with the fact that the effect appears to be distinct from other effects normally associated with nonlinearity and item difficulty, there may indeed be implications here for the detection of violations of local independence within a set of items.

As far as the estimation errors are concerned, these could be evaluated in terms of the relative number of dependent items out of the total number of

$\underline{k}$ test items. Perhaps when $\underline{m}$ is small relative to $\underline{k}$, the seriousness of the estimation error in $\hat{\theta}$ is slight. One of our concerns is in adaptive testing situations where estimates of θ might be obtained on fairly small numbers of items. If local independence is assumed, estimation errors could increase if dependent sequences were used to obtain θ estimates on tests of fairly short lengths.

In our simulations of fixed-length tests, however, where the length of the test was fairly long, it was encouraging to find that in cases of mild dependency, estimation errors were not really much worse than in the independent situation. The effects of the length of the dependent sequences, $\underline{m}$ relative to the total number of items, $\underline{k}$ on the estimation of θ in both fixed-length and adaptive settings needs to be investigated.

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TABLE 1

First Four Eigenvalues From the Principal Component Analyses

Data Set #	ϕ	α	β	m	λ_1	λ_2	λ_3	λ_4
1[a]	.00	1.00	1.00	0	14.23	2.08	1.05	1.00
2[b]	.00	1.00	1.00	0	17.22	.97	.95	.92
3	.52	.40	.60	2	17.17	1.11	.97	.95
4	.48	.60	1.00	6	17.21	1.03	1.01	.95
5	.52/.48	.40/.60	.60/1.00	2/6	17.27	1.08	1.02	1.01
6	1.12	.20	1.00	5	17.24	1.61	.95	.94
7	1.12	.20	.20	3	17.20	1.71	.97	.94
8	1.12/1.12	.20/.20	1.00/.20	5/3	17.32	1.75	1.48	.94
9	1.93	.00	.80	8	17.53	3.17	1.09	.92
10	1.94	.00	.00	6	17.67	3.56	.94	.92
11	1.94/1.93	.00/.00	.00/.80	6/8	18.21	3.80	2.82	1.03
12[c]	1.12	.20	.20	3	13.85	2.18	1.68	1.01

[a]Independent data set with $\underline{b} \sim N(0,1)$

[b]Independent data set with $\underline{b} \sim U(-.25,.25)$

[c]Dependent data set #7 with $\underline{b} \sim N(0,1)$

TABLE 2

Patterns of Component Loadings ≥ |.250|: Independent Case, Data Set #1

Item #	b	Component C_1	C_2
36	-2.037	.346	.326
31	-1.941	.400	.359
34	-1.653	.449	.357
19	-1.592	.395	.321
17	-1.551	.451	.302
47	-1.515	.445	.302
43	-1.485	.459	.300
14	-.775	.580	.266
1	1.989	.344	-.268
32	1.485	.464	-.329
35	1.421	.444	-.323
26	1.374	.471	-.264
28	1.245	.496	-.287
8	1.033	.512	-.285
11	.998	.452	-.251

TABLE 3

Patterns of Component Loadings: Mild Dependency

Data Set #	Item #	Component C_1	C_2	C_3	C_4
3	1	.598	.617	-.033	.180
	2	.572	.661		
4	1	.595	.408	-.033	
	2	.585	.523	-.047	
	3	.599	.401	.129	
	4	.641	.136	.422	
	5	.563	-.066	.526	
	6	.600	-.069	.297	
5	1	.594	-.162	.342	.180
	2	.584	-.188	.440	.223
	3	.600	.134	.240	.299
	4	.641	-.101	-.132	.413
	5	.562	-.099	-.363	.379
	6	.601	-.018	-.238	.186
	30	.619	.558	.160	.090
	31	.610	.566	.103	.134

Note. All remaining items not listed which were not in the dependent sequences loaded ≥ $|.250|$ on C_1 and < $|.250|$ on all other components.

TABLE 4

Patterns of Component Loadings: Medium Dependency

Data Set #	Item #	Component C_1	C_2	C_3	C_4
6	1	.610	.372		
	2	.626	.558		
	3	.619	.587		
	4	.629	.570		
	5	.573	.456		
7	1	.622	.674		
	2	.607	.749		
	3	.606	.699		
8	1	.609	−.298	.228	
	2	.624	−.419	.375	
	3	.618	−.437	.395	
	4	.626	−.433	.377	
	5	.570	−.357	.288	
	30	.637	.515	.419	
	31	.627	.568	.449	
	32	.628	.525	.420	

Note. All remaining items not listed which were not in the dependent sequences loaded $\geq |.250|$ on C_1 and $< |.250|$ on all other components.

TABLE 5

Patterns of Component Loadings: Strong Dependency

Data Set #	Item #	Component C_1	C_2	C_3	C_4
9	1	.649	.286	.509	
	2	.685	.449	.444	
	3	.708	.531	.292	
	4	.705	.586	.142	
	5	.693	.627	-.101	
	6	.667	.645	-.235	
	7	.628	.644	-.321	
	8	.597	.603	-.356	
10	1	.719	.694		
	2	.719	.694		
	3	.719	.694		
	4	.719	.694		
	5	.719	.694		
	6	.719	.694		
11	1	.708	.653	.267	.016
	2	.708	.653	.267	.016
	3	.708	.653	.267	.016
	4	.708	.653	.267	.016
	5	.708	.653	.267	.016
	6	.708	.653	.267	.016
	30	.642	-.240	.107	.532
	31	.689	-.318	.243	.466
	32	.691	-.418	.373	.226
	33	.702	-.423	.441	.064
	34	.699	-.435	.461	-.055
	35	.670	-.397	.522	-.224
	36	.649	-.390	.523	-.281
	37	.623	-.382	.511	-.292

Note. All remaining items not listed which were not in the dependent sequences loaded $\geq |.250|$ on C_1 and $> |.250|$ on all other components.

TABLE 6

C_1 Loadings of Items from Data Set #2
Subsequently Included in Dependent Sequences

Item #	Component C_1
1	.586
2	.588
3	.596
4	.616
5	.552
6	.597
7	.543
8	.573
30	.608
31	.562
32	.601
33	.592
34	.613
35	.566
36	.585
37	.514

Table 7

Patterns of Component Loadings ≥ |.250|: Medium Dependency with b ~ N(0,1)

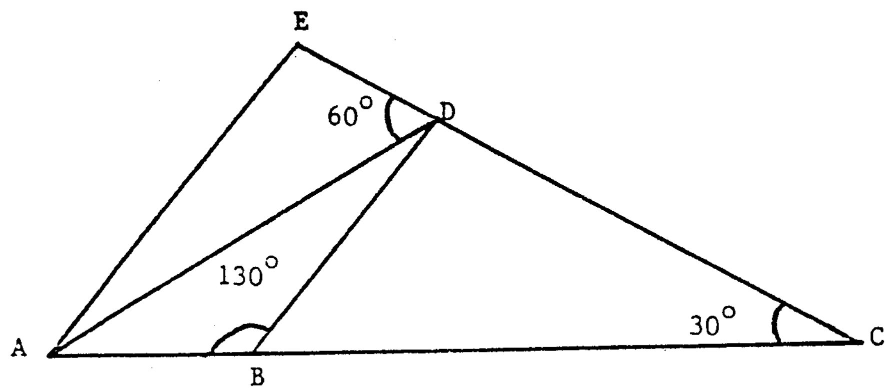
	Item #	b	Component C_1	C_2	C_3
a	1	1.989	.372	.501	.464
	2	-.078	.418	.513	.631
	3	-.730	.400	.422	.629
	36	-2.037	.341	-.298	
	31	-1.941	.395	-.318	
	34	-1.653	.446	-.310	
	19	-1.592	.389	-.314	
	17	-1.551	.447	-.281	
	47	-1.515	.441	-.266	
	43	-1.485	.457	-.281	
	32	1.485	.470	.292	
	35	1.421	.448	.263	

[a]Dependent item sequence

TABLE 8

Bias and Variability Statistics for Estimates of θ (LOGIST 5)

Data Set #	$E\{\hat{\theta} - \theta\}$	$\hat{\sigma}(\hat{\theta})$	Maximum Overestimation	Maximum Underestimation
2	-.009	.235	.865	-1.054
3	.001	.241	.945	-1.114
4	-.025	.246	.945	-1.114
5	-.029	.248	.945	-1.114
6	-.051	.257	.945	-1.365
7	-.003	.251	1.155	-1.114
8	-.051	.263	.945	-1.365
9	-.130	.280	1.155	-1.645
10	-.017	.286	1.155	-1.465
11	-.167	.325	1.155	-1.865



Given: $\overline{AE}$ is parallel to $\overline{BD}$

$\angle$ ABD = 130°

$\angle$ EDA = 60°

$\angle$ BCD = 30°

1. What is $\angle$ CBD?

   A. 50°
   B. 60°
   C. 65°
   D. 100°

2. Find $\angle$ BDC.

   A. 130°
   B. 100°
   C. 90°
   D. 85°

3. Find $\angle$ EAD.

   A. 20°
   B. 30°
   C. 35°
   D. 60°

Figure 1. Example Test Items which Illustrate Local Dependence.

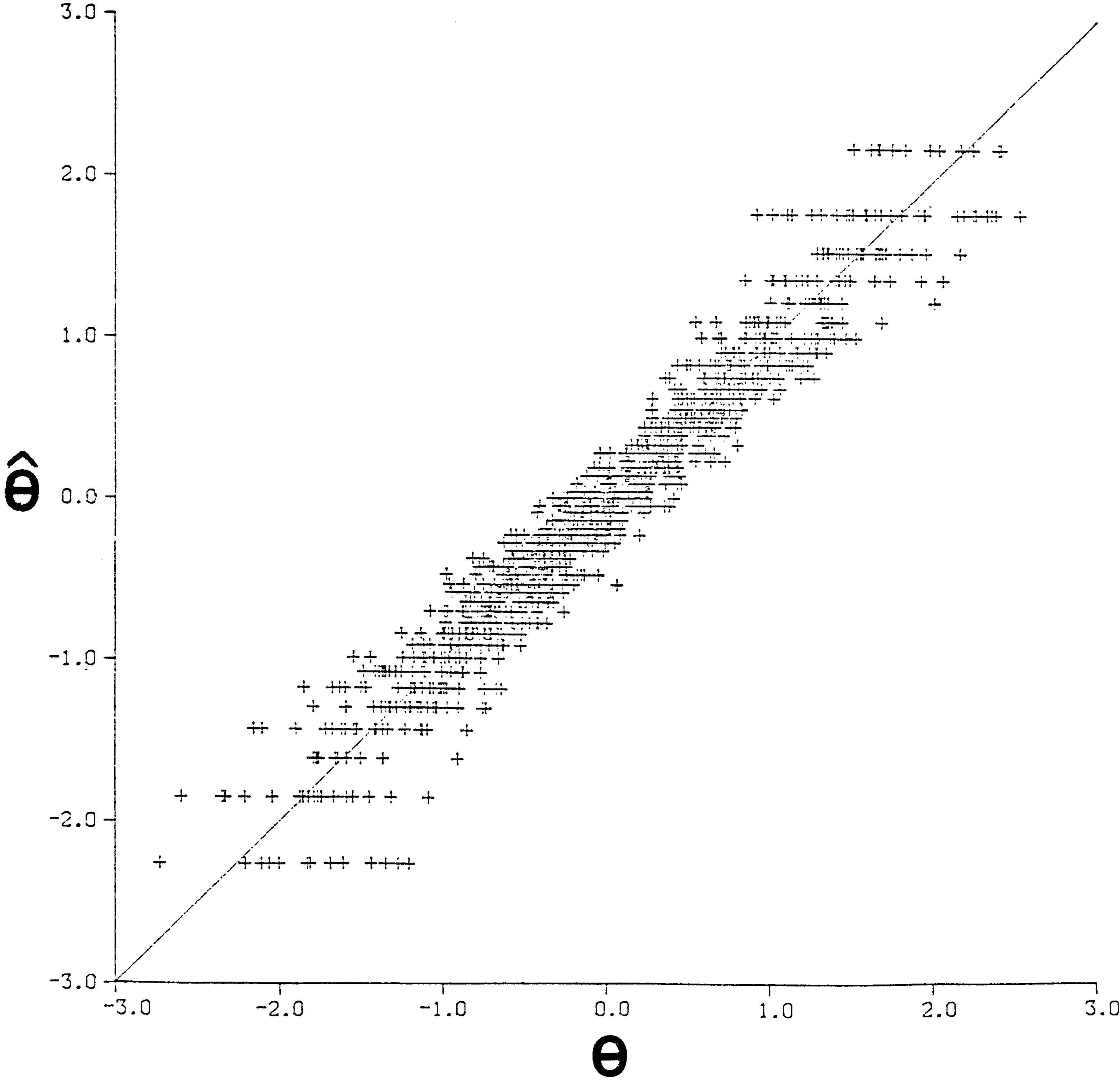


Figure 2. θ vs. $\hat{\theta}$ for Data Set #2.

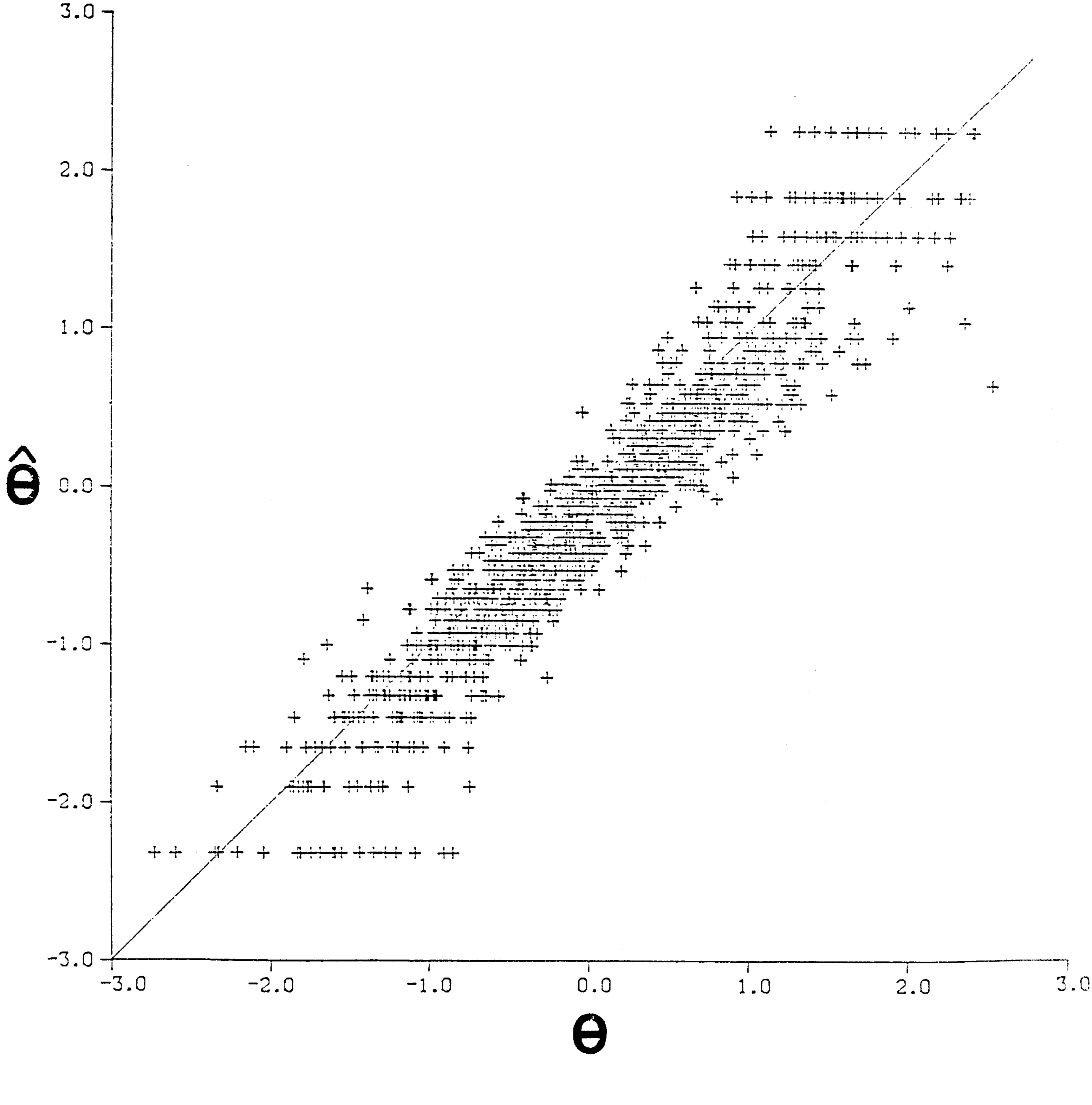


Figure 3. θ vs. $\hat{\theta}$ for Data Set #11.